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A First Look at Econometric Policy Evaluation

The models used in the preceding chapter to introduce the methods of policy evaluation are highly stylized. The parameters are assumed rather than estimated, and the shocks to the equations are not empirically based. Hence, they are not of much use for practical policy evaluation. Fully estimated, economywide models that are simple enough to use for illustration are still hard to come by. In this chapter, I examine one such model—a fully estimated rational expectations model for the United States—and I take a first look at econometric policy evaluation.

This model is a useful prelude to the multicountry model introduced in Chapter 3, although in many respects it is still rudimentary. However, for the purposes of illustrating the econometric techniques and the policy methods, a rudimentary model offers important advantages. The model can be kept linear and small. Algebraic formulas rather than numerical solution algorithms can be employed. Systemwide estimation techniques are feasible. The cross-equation constraints, which are the hallmark of the rational expectations approach to policy evaluation, are easy to illustrate algebraically. These constraints become internal to the numerical computations, and hence they are largely invisible in larger, more realistic models.

The single-country model I present in this chapter places considerable emphasis on the institutional detail of wage and price setting. In fact, the wage and price sector looms large and tends to dominate the rest of the model. Wage and price setting is responsible for much of the dynamics in the model. These wage-price dynamics produce cyclical swings in output, employment, and inflation that closely resemble business-cycle fluctuations in the United States. In a typical cyclical pattern, inflation accelerates, government policy becomes restrictive, recession ensues, policy eases, a renewed inflationary boom begins, and so on.

The criterion used for evaluating policy is the size of these recurrent swings of inflation and employment. Good policy rules hold these swings within tolerable limits on the average. An anti-inflation policy, for example, would be judged not only by its forecasted success at bringing down inflation from a historically given high level but also by its ability to prevent renewed cyclical inflationary surges. Similarly, an antirecession policy would be judged not only by its success at stimulating the economy out of a particular recession but also by the attention it pays to expectations and the cyclical workings of the economy to prevent a renewed recession shortly thereafter.

The premise upon which the wage and price sector of this econometric model is based is that forecasts of future inflation rates and business conditions, which figure into wage negotiations, can actually be represented by the forecasts of the model itself. This is, of course, the rational expectations assumption empirically at work. As a technique it is useful if it works better than available alternative techniques. Practical alternative expectations techniques that have been used in traditional econometric models since the 1950s include adaptive expectations mechanisms or subjective "constant adjustment" of expectations equations to make them look more reasonable or perhaps consistent with forecasters' expectations. The view presented here is that the rational expectations technique is empirically more useful than these alternatives. Of course, future research may discover alternative techniques (perhaps with learning behavior incorporated explicitly) that may be superior to those currently available.

The chapter is organized as follows. First, I describe the structure of the model. Second, I develop an estimation technique and describe the estimation of the model over a historical sample period. Third, I use the model for econometric policy evaluation.

2.1 The Structure of the Model

The macroeconomic model describes the behavior of five endogenous variables in the United States: the average wage, the average price, employment, output, and the money supply. Determination of employment, output, and money supply is rather straightforward, given the determination of wages and prices. Hence I begin with the wage-price sector, which also plays an important role in the multicountry model discussed in Chapter 3.

The Determination of Wages and Prices

A difficulty with trying to incorporate wage setting into an empirical model is the reduction of the intricate details of real-world wage data into a manageable framework suitable for aggregate-data analysis without losing those details that make wage setting important. The trick is to find a method of aggregating across individual wage contracts set in the same period for the same length of time and then to determine the behavior of these "contract-wage aggregates." If the method is to capture the interaction between wage contracts negotiated at different dates, then aggregation across wages set at different points in time can only occur at the last stage of analysis, after this interaction has been modeled. The problem becomes more difficult when the individual contract wages are unobservable.

Although very little information is available on implicit or explicit wage contracts in the United States, information published by The Bureau of Labor Statistics (BLS) on the approximately 10 percent of U.S. workers engaged in major collective bargaining gives perspective to the problem. In Chapter 7 I use these data (see Table 7-1) on the number of workers negotiating explicit contracts of different lengths each quarter to calibrate a wage model. For now it is enough to say that the data show that it would be too gross an approximation to assume that contracts in the U.S. economy are all of the same length (as in the stylized model of Chapter 1). Even if we ignore the 90 percent of all workers not represented by major unions—and who probably work under implicit or explicit contracts averaging about one year in duration—the range of contract length is quite wide. On the other hand, abstracting from seasonal influences, the distribution of workers by contract length does not show any systematic pattern or tendency to change over the business cycle. Hence, if one is ultimately interested in describing the behavior of seasonally adjusted data, and if one-year and two-year contracts are more representative of the economy as a whole, then a first approximation would be to assume that the distribution of workers by contract length is homogeneous over time. This approximation results in a major simplification for the aggregation procedures and will be made in the analysis that follows.

To complete the aggregation procedure, three additional approximations need to be made: the variation in average contract wages across contract classes of different lengths is assumed to be negligible relative to the variation in contract wages over time; all wage adjustments are assumed to occur during the quarter in which the contract is negotiated; any indexing that changes the wage contract at regular intervals during the contract period can be approximated as a series of short-term contracts, rather than as one long-term contract. The last approximation is partially a matter of definition and will tend to make our estimated distribution of contract lengths shorter than what a literal reading of the BLS data would indicate. Most indexing in the U.S. economy is found in multiyear contracts. The import of this approximation is that these indexed contracts are comparable to shorter contracts with lengths equal to the indexing review period. It is an approximation because contract-wage adjustments are influenced by a wider range of factors when they are adjusted by renegotiation rather than by indexing.

The starting point for implementing these approximations is the staggered wage-setting model used in Taylor (1980) and used in the model of

Section 1.4 of Chapter 1. In the standard, nonsynchronized version of this model, only a small fraction of workers have their contract wage changed in a given time period. The contract wage is assumed to be set to equal the expected average wage in the economy during the upcoming contract period, plus an amount that depends on expected excess demand in the economy as measured by the deviations of actual output from trend output over the next four quarters. The crucial parameter to estimate in the model is the sensitivity of the wages to this future excess-demand term.

In my 1980 paper, I considered the simple example in which 25 percent of workers change wages each quarter with the wage being set for one year. This example seemed to work well as approximation in that certain general features of the dynamic behavior of wages in the United States could be explained by the model. However, for detailed empirical work, one needs to go beyond this simple example.

Nonsynchronized Wage Setting with Different Contract Lengths

In the more general, but still nonsynchronized version of the model, not all workers are working under contracts that last the same number of quarters (the synchronized version is discussed in Chapter 3). The "contract" wage is determined according to the following equation:

$$\begin{aligned} x_t = {} & \pi_0 E w_t + \pi_1 E w_{t+1} + \pi_2 E w_{t+2} + \pi_3 E w_{t+3} \\ & + h(\pi_0 E e_t + \pi_1 E e_{t+1} + \pi_2 E e_{t+2} + \pi_3 E e_{t+3}) + v_t, \end{aligned} \tag{2.1}$$

where x_t is the log of the contract wage, w_t is the log of the average wage, e_t is an index of excess demand in the labor market, and v_t is a disturbance term. The symbol E refers to the conditional-expectation operator based on information through time t. The aggregate wage is given by the equation[1]

$$w_t = \pi_0 x_t + \pi_1 x_{t-1} + \pi_2 x_{t-2} + \pi_3 x_{t-3}. \tag{2.2}$$

Interpreting the π-Coefficients

As described above, in the simple staggered contract model of Taylor (1980), the π-parameters were set to equal .25 with the interpretation that 25 percent of all workers sign contracts each quarter and that each contract lasts four quarters. We now must consider the interpretation of these parameters in the more general case. We seek an interpretation in terms of the

[1]For the case where $\pi_i = .25$ the arguments used in Taylor (1980) lead to the weights in Equation (2.1) being identical to those in Equation (2.2). An extension of these arguments can be used in the more general case of the π-weights. However, alternative wage-contract equations can be derived, in which the weights on the future wages and output levels are not the same as the π-weights.

distribution of workers by different lengths of contracts. This interpretation is also used when we consider synchronized contracting. Let

x_{jt} = average contract wage set in quarter t in contracts that are j quarters in length $(j = 1, \ldots, J)$;
n_{jt} = number of workers affected by contract-wage changes in quarter t in contracts that are j quarters in length $(j = 1, \ldots, J)$;
f_{jt} = fraction of workers in quarter t affected by contract-wage changes in contracts that are j quarters in length $(j = 1, \ldots, J)$;
a_{jt} = fraction of workers in the labor force in quarter t who have contracts of length j $(j = 1, \ldots, J)$;
w_t = average wage in the economy in quarter t.

Then, by definition of f_{jt}, a_{jt}, and w_t we have

$$f_{jt} = \frac{n_{jt}}{\sum_{j=1}^{J} n_{jt}} \tag{2.3}$$

$$a_{jt} = \frac{\sum_{s=0}^{j-1} n_{jt-s}}{\sum_{j=1}^{J}\sum_{s=0}^{j-1} n_{jt-s}} \tag{2.4}$$

$$w_t = \frac{\sum_{j=1}^{J}\sum_{s=0}^{j-1} n_{jt-s}x_{jt-s}}{\sum_{j=1}^{J}\sum_{s=0}^{j-1} n_{jt-s}} \tag{2.5}$$

If the distribution of workers by contract length is homogenous over time $(n_{jt} = n_j)$ and if the variation of average contract wages over contracts of different length is negligible $(x_{jt} = x_t)$, then (2.5) reduces to

$$\begin{aligned} w_t &= \frac{\sum_{j=1}^{J}\sum_{s=0}^{j-1} n_j x_{t-s}}{\sum_{j=1}^{J}\sum_{s=0}^{j-1} n_j} \\ &= \frac{\sum_{s=0}^{J-1}\sum_{j=s+1}^{J} n_j x_{t-s}}{\sum_{j=1}^{J} j n_j} \\ &= \sum_{s=0}^{J-1} \pi_s x_{t-s}, \end{aligned} \tag{2.6}$$

where the π_s are defined as

$$\pi_s = \left(\sum_{j=s}^{J-1} n_{j+1}\right)\left(\sum_{j=1}^{J} j n_j\right)^{-1}. \tag{2.7}$$

Note that the π-weights sum to 1 and are time invariant. Hence the aggregate wage w_t is a moving average of the "index" of contract wages x_t set in the recent past. The π-weights can also be written in terms of the $a_{jt} = a_j$. For example, when $J = 4$,

$$\pi_0 = a_1 + a_2/2 + a_3/3 + a_4/4 \tag{2.8}$$

$$\pi_1 = a_2/2 + a_3/3 + a_4/4 \tag{2.9}$$

$$\pi_2 = a_3/3 + a_4/4 \tag{2.10}$$

$$\pi_3 = a_4/4. \tag{2.11}$$

Some examples are useful for illustrating how the π-weights depend on the distribution of workers across contracts of different lengths. If all contracts are the same length, say four quarters, then $n_1 = n_2 = n_3 = 0$, and $\pi_0 = \pi_1 = \pi_2 = \pi_3 = .25$. This is the type of contract distribution used in the theoretical examination of staggered contracts presented in Taylor (1980). If the distribution of workers across contracts of different lengths in a given quarter is uniform up to four quarter contracts, then $n_1 = n_2 = n_3 = n_4$, and the π-weights decline linearly: $\pi_0 = .4$, $\pi_1 = .3$, $\pi_2 = .2$, and $\pi_3 = .1$. Note that the distribution of workers across contracts can be recovered from the π-weights through the identities

$$(\pi_{i-1} - \pi_i)\pi_0^{-1} = f_i \qquad i = 1, 2, \ldots, J, \quad (\pi_J = 0), \tag{2.12}$$

$$(\pi_{i-1} - \pi_i)i = a_i \qquad i = 1, 2, \ldots, J, \quad (\pi_J = 0). \tag{2.13}$$

The π-weights, and hence this distribution of workers, are part of the economic structure to be estimated. In the estimation we can use assumptions about the a_i to impose constraints on the π_i. For example, we must have that $a_i > 0$ and that a_i sum to 1.

The aggregate wage w_t evolves from the index of contract wages x_t. Since the contracts that constitute this index will prevail for several quarters, workers and firms negotiating a contract wage will be concerned with the labor market conditions expected to prevail during the upcoming contract period. For example, those setting four quarter contracts must forecast labor market conditions four quarters ahead. Moreover, in the process of forecasting future wages, these firms and workers will take account of contracts negotiated in the recent past, as these will be part of the relative wage structure during part of the contract period. Equation (2.1) is a behavioral equation for the determination of the contract-wage index that takes account of these factors.

Some of the important questions about wage and price dynamics can be cast in terms of the parameters in Equation (2.1). The parameter h should be positive. Whether h is large or small is relevant for determining how accommodative policy should be toward price or supply shocks. However, lagged price shocks could enter Equation (2.1) directly to portray catch-up effects. This latter possibility will be considered below when we introduce a stochastic structure to the behavioral equations.

The excess-labor demand variable e in Equation (2.1) will be measured by the unemployment rate (with the sign reversed). Hence, I do not take a logarithmic transformation of e_t. In Equation (2.1), and in the other equations of the model, the constant terms and any trend factors will be omitted since I will be working with detrended data when the model is estimated.

Given the aggregate wage w_t as determined from the contract-wage index x_t, I will assume that prices are determined on the basis of wage and other costs, that is,

$$p_t = \hat{w}_t + \theta_p(L)\, u_{pt}, \tag{2.14}$$

where $\theta_p(L)$ is a lag polynomial and u_{pt} is a serially uncorrelated shock. The term $\theta_p(L)\, u_{pt}$ is a measure of other factors affecting pricing decisions. Our assumption is that the prices that underlie index p_t are relatively free to vary, so that no additional dynamics in the model enter explicitly through staggered price contracts. However, we do model the influence of wages on price decisions as operating with a one period lag; firms forecast their wage costs w_t during the current period and set p_t accordingly. In the empirical work, the error term will be a general stochastic process, so that exogenous serial correlation in the detrended real wage $w_t - p_t$ will be part of the model. Some of the other factors that may affect p_t relative to w_t might be demand conditions, raw material costs, and temporary fluctuations in productivity about trend.

Aggregate Demand and Employment

The remaining parts of the model are rudimentary, especially in comparison with the wage-price sector. An aggregate-demand equation is given by

$$y_t = \alpha_1(m_t - p_t) + \theta_y(L)\, u_{yt}, \tag{2.15}$$

where y_t is the log of (detrended) real output, and m_t is the log of the (detrended) money supply. As in Equation (1.7), $\theta_y(L)$ is a lag polynomial, and u_{yt} is a serially uncorrelated error. Missing from Equation (2.15) is a measure of the real interest rate that would link this equation explicitly with investment and consumption decisions and with fiscal policy. Also missing are explicit adjustment terms to reflect lags in the impact of monetary policy on real output. Our approach in this “intermediate” model is to think of these factors as part of a general stochastic structure; a better procedure would be to incorporate lagged values of y_t, m_t, and p_t into Equation (2.15), along with measures of the real interest rate. In fact, this is what is done in the model introduced in Chapter 3.

To link the aggregate-demand variable y_t to our measure of labor-market tightness e_t, we will utilize an Okun’s law type of relationship with serial correlation to approximate temporary discrepancies or lags. That is,

$$e_t = \alpha_2 y_t + \theta_e(L)\, u_{et}, \tag{2.16}$$

where $\theta_e(L)$ is a polynomial in the lag operator and u_{et} is a serially uncorrelated error. Since y_t is the log deviation of output about trend, it will behave like the negative of the percentage output gap. With e_t defined as the negative of the unemployment rate, α_2 should approximately equal the inverse of Okun's law multiplier.

The Monetary Policy Rule

Since fiscal policy is assumed to be incorporated in the error structure of the aggregate-demand equation (2.15), the only tool of aggregate-demand management that is explicitly modeled is monetary policy. Consider feedback reaction functions of the following form:

$$m_t = g_1\hat{p}_t + g_2\hat{w}_t + g_3\hat{y}_t + \theta_m(L)\,u_{mt}, \tag{2.17}$$

where $\theta_m(L)$ is a polynomial in the lag operator and u_{mt} is serially uncorrelated. Equation (2.17) is a feedback rule because all the variables on the right-hand side are predetermined; they are forecasts of conditions in period t given information through the previous period. Coefficient g_3 represents attempts at countercyclical monetary policy; we would expect g_3 to be negative. Coefficients g_1 and g_2 and their sum are measures of how accommodative monetary policy is to price shocks or wage shocks. If $g_1 = g_2 = 0$, then policy is not accommodative at all, whereas if $g_1 < g_2$, then policy is less accommodative to price shocks than to wage shocks. An important policy question is whether it is appropriate to accommodate prices, which tend to be more volatile, but not wages, which are indicators of the underlying inflation rate; the answer depends in part on whether prices enter the wage equation. In order to explore possible variations in g_1 and g_2, it is necessary to estimate these parameters jointly with the rest of the model. It should be emphasized that the form of Equation (2.17) is not derived from a policy-optimization procedure. Later on in Section 2.5, I consider the optimal choice for the g parameters. However, in general, an optimal monetary policy would depend on lagged values of the endogenous variables or on the shocks to the other equations.

Summary of the Equations and the Stochastic Structure

Gathering the above equations together we have

$$\begin{aligned}
y_t &= \alpha_1(m_t - p_t) + \theta_y(L)\,u_{yt};\\
p_t &= \hat{w}_t + \theta_p(L)\,u_{pt};\\
m_t &= g_1\hat{p}_t + g_2\hat{w}_t + g_3\hat{y}_t + \theta_m(L)\,u_{mt};\\
e_t &= \alpha_2 y_t + \theta_e(L)\,u_{et};\\
w_t &= \pi(L)\,x_t;\\
x_t &= \pi(L^{-1})\,\hat{w}_t + h\pi(L^{-1})\,e_t + \theta_{xp}(L)\,u_{pt} + \theta_x(L)\,u_{xt}.
\end{aligned} \tag{2.18}$$

Note that in the contract-wage index equation, I have added an error structure consisting of $\theta_{xp}(L)\,u_{pt}$, which captures catch-up effects from past price shocks to wages, and $\theta_x(L)\,u_{xt}$, which allows for the wage shocks to be serially correlated.

I will assume that the vector $(u_{yt}, u_{pt}, u_{mt}, u_{et}, u_{xt})$ is serially uncorrelated with zero mean and covariance matrix Ω. This correlation assumption does put restrictions on the model despite the fact that I am considering fairly general error processes in each equation via the θ-parameters. I am assuming that there is only one cross-effect in the errors (θ_{xp}); the omission of other cross-effects is a constraint.

The parameters of the model are α_1, α_2, h, g_1, g_2, g_3, Ω, and the coefficients of the polynomials $\pi(L)$, $\theta_y(L)$, $\theta_p(L)$, $\theta_m(L)$, $\theta_e(L)$, $\theta_{xp}(L)$, $\theta_x(L)$. Hence, the number of parameters depends on the length of the longest contract considered (which determines the order of π) and on the extent of serial correlation. The model offers two simultaneous equations where more than one current endogenous variable appear. Most of the equations contain one-period-ahead rational forecasts of the endogenous variables, but only the contract-wage equation contains multiperiod forecasts. The multicountry model in Chapter 3 considers multiperiod forecasts in other equations. In Section 2.2 I show how the model can be manipulated to obtain a form that can be estimated.

2.2 Solution and Estimation Techniques

In this section I show how the model can be represented as a five-dimensional linear vector autoregressive moving average (VARMA) system with cross-equation constraints. This constrained linear system can then be estimated by full-information maximum-likelihood techniques, which are still not tractable in the larger nonlinear systems described in later chapters. The essence of the rational expectations approach is that the cross-equation constraints must be fully specified when doing policy analysis. The maximum-likelihood approach is desirable because the constraint is imposed at the time of estimation and hence tested along with other features of the model.

Solving the Model: Obtaining a Linear Vector Autoregression

The contract-wage equation for x_t involves forecasts of the wage rate w_t and of the labor market demand e_t as far into the future as the length of the longest contract. These forecasts are conditional on all information available through the end of period $t-1$, and can be written as functions of the past shocks to each equation of the model. The solution technique is to solve the equations of the system for the rational forecasts of the wage rate and of the labor-market variable, to substitute these into the contract-wage equation, and finally to determine a reduced form for the contract wage.

First, by taking expectations on both sides of Equation (2.18), the solutions for the forecasts of the labor market variable and of the wage rate can be shown to be

$$\begin{aligned}\hat{e}_t &= -\alpha_2(1-\alpha_1 g_3)^{-1}[\beta\alpha_1\pi(L)\hat{x}_t - \theta_y(L)\hat{u}_{yt} + \beta_1\alpha_1\theta_p(L)\hat{u}_{pt} \\ &\quad - \alpha_1\theta_m(L)\hat{u}_{mt}] + \theta_e(L)\hat{u}_{et} \qquad (2.19)\\ \hat{w}_t &= \pi(L)\hat{x}_t, \qquad (2.20)\end{aligned}$$

where $\beta = 1 - g_1 - g_2$ and $\beta_1 = 1 - g_1$, and where it should be noted that $\hat{u}_{ys} = 0$ for $s > t-1$ and $\hat{u}_{ys} = u_{ys}$ for $s \leqslant t-1$ and similarly for the other random shocks.

Substituting Equations (2.19) and (2.20) into the equation for the contract wage x_t and taking expectations results in

$$\begin{aligned}&[1-(1-h\gamma\beta\alpha_1)\pi(L^{-1})\pi(L)]\hat{x}_t \\ &\quad = h\gamma\pi(L^{-1})\theta_y(L)\hat{u}_{yt} - h\gamma\beta_1\alpha_1\pi(L^{-1})\theta_p(L)\hat{u}_{pt} \\ &\qquad + h\gamma\alpha_1\pi(L^{-1})\theta_m(L)\hat{u}_{mt} + h\pi(L^{-1})\theta_e(L)\hat{u}_{et} \\ &\qquad + \theta_{xp}(L)\hat{u}_{pt} + \theta_x(L)\hat{u}_{xt}, \qquad (2.21)\end{aligned}$$

where $\gamma = \alpha_2(1-\alpha_1 g_3)^{-1}$. Equation (2.21) is a difference equation in the forecast of the contract wage $\hat{x}_{t+s}$ conditional on information through period $t-1$ with various combinations of the past shocks as forcing variables. To solve the equation, we note its symmetry: the coefficients of L^s and L^{-s} in $\pi(L^{-1})\pi(L)$ are the same. Hence, the lag operator in brackets on the left-hand side of (2.21) can be factored into a form $\lambda R(L)R(L^{-1})$, as explained by Hansen and Sargent (1980). Imposing stability on the system requires that $R(L)$ be chosen so that its roots are outside or on the unit circle. Multiplying both sides of (2.21) through by $[\lambda R(L^{-1})]^{-1}$ and adding u_{xt} to the equation gives

$$\begin{aligned}R(L)x_t &= h\gamma H_y(L)u_{yt} + [H_{xp}(L) - h\gamma\beta_1\alpha_1 H_p(L)]u_{pt} \\ &\quad + h\gamma\alpha_1 H_m(L)u_{mt} + hH_e(L)u_{et} + H_x(L)u_{xt} + u_{xt}, \qquad (2.22)\end{aligned}$$

where

$$\begin{aligned}H_y(L) &= [(\lambda R(L^{-1}))^{-1}\pi(L^{-1})\theta_y(L)]_+ \\ H_p(L) &= [(\lambda R(L^{-1}))^{-1}\pi(L^{-1})\theta_p(L)]_+ \\ H_m(L) &= [(\lambda R(L^{-1}))^{-1}\pi(L^{-1})\theta_m(L)]_+ \\ H_e(L) &= [(\lambda R(L^{-1}))^{-1}\pi(L^{-1})\theta_e(L)]_+ \\ H_x(L) &= [(\lambda R(L^{-1}))^{-1}\theta_x(L)]_+ \\ H_{xp}(L) &= [\lambda(R(L^{-1}))^{-1}\theta_{xp}(L)]_+ \qquad (2.23)\end{aligned}$$

and the notation $[\ ]_+$ means that only the positive powers of L in the polynomial products are retained.

Equation (2.22) is autoregressive in the contract wage with moving average errors entering from all the equations of the model. Past shocks to aggregate demand enter the equation with positive coefficients (if these shocks are positively correlated), because aggregate-demand shocks are an indicator of low unemployment in the near future that tends to bid up contract wages ($h > 0$). For similar reasons, monetary surprises u_{mt} and employment surprises u_{et} enter the wage equation with positive coefficients. However, the impact of past price shocks on wage determination is ambiguous. We would expect the sum of the coefficients of $H_{xp}(L)$ to be positive, but this catch-up effect may be offset in the reduced form by $h\gamma\beta_1\alpha_1 H_p(L)$, which captures the "anti-inflation" reaction of the monetary authorities to price shocks. If $g_1 = 1$, so that price movements are completely accommodated, then $\beta_1 = 0$ and the "anti-inflation" effect drops out. But if $g_1 < 1$, then price shocks may appear to have a negative effect on wages.

The sensitivity of wages to excess demand h enters into the reduced-form wage equation in several ways. It is one of the determinants of the autoregressive coefficients in $R(L)$ because it appears in the symmetric lag polynomial on the left-hand side of Equation (2.21). Higher values of h will tend to reduce the coefficients of $R(L)$ and make wage changes less persistent. However, h also enters into the serial correlation and cross-serial correlation coefficients in the x_t equation. In these serial correlation expressions, higher values of h will raise the impact of all these shocks on wage behavior. This effect represents the interaction between the forecasts of future labor-market demand (via extrapolations from the model using recent observations) and the impact of demand on wage behavior.

Policy parameters g_1, g_2, and g_3 also enter the equation in several ways. The sum of $g_1 + g_2$ represents the combined accommodation of monetary policy to price and wage shocks. This sum enters the autoregressive coefficients through parameter β. Larger values of $g_1 + g_2$ imply larger autoregressive coefficients and more persistence of wage changes. As mentioned, g_1 produces an effect that g_2 does not: the price-accommodation parameter tends to affect the feedback of prices onto wage determination. Hence, the size of g_1 bears on the impact of price shocks on the wage-price dynamics. But g_1 does not have any unique ability to change the propagation of these price shocks once they are into the dynamics. Both accommodation parameters are equally powerful at changing the propagation properties (i.e., the autoregressive weights). Proposals for policies that are very accommodative toward price shocks, but not toward wage shocks, evidently place emphasis on reducing the propagation effects while ignoring temporary impulse effects.

To complete the solution of the model, we need to substitute the reduced-form contract equation back into the structural equations. First, we compute the average wage w_t, which simply requires us to pass x_t through the moving average operator $\pi(L)$. This results in

$$\begin{aligned} w_t = {} & -R_1(L)w_t + G_y(L)u_{yt} + G_p(L)u_{pt} + G_m(L)u_{mt} \\ & + G_e(L)u_{et} + G_x(L)u_{xt} + u_{xt}, \end{aligned} \tag{2.24}$$

where

$$\begin{aligned}
G_y(L) &= \pi(L)\,h\gamma\lambda^{-1}H_y(L)\\
G_p(L) &= \pi(L)\,[H_{xp}(L) - h\gamma\beta_1\alpha_1 H_p(L)]\\
G_m(L) &= \pi(L)\,[h\gamma\alpha_1\lambda^{-1}H_e(L)]\\
G_e(L) &= \pi(L)\,[h\lambda^{-1}H_m(L)]\\
G_x(L) &= \pi(L)\,H_x(L)\\
R_1(L) &= [R(L)]_+ .
\end{aligned}$$

Equation (2.24), along with the equations for y_t, p_t, m_t, and e_t, constitute a system in which only one-period-ahead forecasts of the endogenous variables appear, all with viewpoint date $t-1$. Using matrix notation, we can write this system as

$$z_t = C_0 z_t + C_1 \hat{z}_t + C(L) z_t + D(L)\,u_t + u_t, \tag{2.25}$$

where

$$\begin{aligned}
z_t &= (y_t, p_t, m_t, e_t, w_t)'\\
u_t &= (u_{yt}, u_{pt}, u_{mt}, u_{et}, u_{xt})'
\end{aligned}$$

and where the relatively sparse C and D matrices are

$$C_0 = \begin{bmatrix} 0 & -\alpha_1 & \alpha_1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ \alpha_2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ g_3 & g_1 & 0 & 0 & g_2\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C(L) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -R_1(L) \end{bmatrix}$$

$$D(L) = \begin{bmatrix} \theta_y(L) & 0 & 0 & 0 & 0\\ 0 & \theta_p(L) & 0 & 0 & 0\\ 0 & 0 & \theta_m(L) & 0 & 0\\ 0 & 0 & 0 & \theta_e(L) & 0\\ G_y(L) & G_p(L) & G_m(L) & G_e(L) & G_x(L) \end{bmatrix}.$$

Substituting for $\hat{z}_t$ by taking expectations on both sides of Equation (2.25) results in

$$z_t = C_0 z_t + (I - C_0)(I - C_0 - C_1)^{-1} C(L) z_t + (I - C_0)(I - C_0 - C_1)^{-1} D(L) u_t + u_t \qquad (2.26)$$

or

$$A_0 z_t = A(L) z_t + B(L) u_t, \qquad (2.27)$$

where the matrix A_0 and the matrix polynomials $A(L)$ and $B(L)$ are defined accordingly. This equation system is simultaneous ($C_0 \neq 0$) with an autoregressive moving average (ARMA) structure. The structure is very heavily constrained both because $C(L)$ and $D(L)$ are constrained as discussed and because C_0 and C_1 contain many of the same elements that are in $C(L)$ and $D(L)$.

Maximum-Likelihood Estimation

The system of Equation (2.26) can be estimated with maximum-likelihood techniques using the working assumption that u_t is normally distributed. The concentrated log-likelihood function (given a set of initial conditions) can be written as

$$\frac{T}{2} \log \left| \sum_{t=1}^{T} u_t u_t' \right| + T \log |I - C_0|, \qquad (2.28)$$

excluding the constant term. This function can be evaluated numerically in terms of the fundamental structural and stochastic parameters introduced in Section 2.1, and hence it can be maximized by using numerical techniques. Tests of the model can be developed using likelihood-ratio tests, and standard errors can be estimated from the matrix of second derivatives of the likelihood function. Because the factorization technique requires finding the roots of polynomials with orders as high as 8, the constrained-likelihood function cannot be represented analytically. Hence, derivatives and second derivatives must be computed numerically.[2] The matrix of second derivatives was computed at the last iteration for the purposes of estimating the variance-covariance matrix of the estimated coefficients.

2.3 Empirical Implementation and Parameter Estimates

Specific empirical measures for the five endogenous variables correspond to the seasonally adjusted real GNP for y, the GNP deflator for p, the compensation per man hour in the private sector for w, the $M1$ definition

[2] I had the most success using the Davidon-Fletcher-Powell technique, computing numerical first derivatives during each iteration. See Goldfeld and Quandt (1972) for a description of this numerical technique.

of the money supply for m, and the (inversely scaled) unemployment rate for males between the ages of 25 and 54 for e. This single-country model was originally estimated for the United States over the sample period 1960:1 through 1977:4, and I focus on the same sample period here. (The U.S. part of the multicountry model described in Chapter 3 is estimated with more recent data.) Using the original parameter estimates permits an assessment of the model as a method to evaluate alternative policy proposals to end the high inflation of the late 1970s and early 1980s. For all the variables except e, a logarithmic transformation was used. The logarithmically transformed data were then detrended by a regression on a linear-time trend over the sample period.

In order to limit the number of parameters to be estimated in the stochastic processes describing the shocks to each equation, the general moving average representation for these shocks was restricted. The restrictions were of two types: first, the moving average was truncated after a certain number of lags, and second, the coefficients of the resulting truncated lag were constrained to be functions of a smaller number of parameters than the length of the lag. More specifically, the following parametric forms were assumed for the θ-polynomials describing the stochastic part of the model

$$\theta_y(L) = [1 - \rho_{y1}L - \rho_{y2}L^2]^{-1} \tag{2.29}$$

$$\theta_p(L) = [1 - \rho_p L]^{-1} \tag{2.30}$$

$$\theta_m(L) = [1 - \rho_{m1}L - \rho_{m2}L^2]^{-1} \tag{2.31}$$

$$\theta_e(L) = [1 - \rho_{e1}L]^{-1} \tag{2.32}$$

$$\theta_{xp}(L) = [1 - \rho_{xp}L]^{-1} \tag{2.33}$$

$$\theta_x(L) = 1 + \theta_{x1}L + \theta_{x2}L^2 + \theta_{x3}L^3 + \theta_{x4}L^4, \tag{2.34}$$

where the subscripted ρ and θ were treated as unconstrained. The infinite-power series in the lag operator in Equations (2.29) through (2.33) were truncated at the fourth order. We found that some truncation of these polynomials was necessary to keep the order of the moving average parts of the vector model from growing too large. The G-polynomials in Equation (2.24) have orders equal to the maximum contract length plus the order of the corresponding θ-polynomials. (Because of the expectations variables, operation on both sides of the equation by the inverse of the nontruncated θ-polynomials was not useful in reducing the moving average length as is typical in ARMA modeling.) In choosing the parametric form and the truncations in (2.29) through (2.34), the serial correlation matrices of the residuals were examined; when the serial correlation was judged to be too high, the restrictions were loosened.

In addition to restricting the θ-polynomials, I also put constraints on the π-polynomial that describes the distribution of contracts in the economy. For the results reported, I truncated π at the seventh lag, thereby permitting a maximum contract length of eight quarters. The shape of π was also constrained to decline very slowly for short lags and never to take on negative values. (Recall that neither negative nor increasing π-weights makes any economic sense from the point of view of contract distributions.) Operationally, these constraints were imposed by assuming that π_j/π_0 is equal to points on the right-hand side of a normal-density function in which the "standard deviation" parameter was estimated. The hypothesis can be tested by estimating the model with fully unconstrained π-weights and by comparing the value of the likelihood function with that in the constrained case.

With these specifications the system (2.27) becomes a five-dimensional VARMA $(7, 11)$ model with simultaneous relationships among the dependent variables. The ninety elements of the ARMA matrices are functions of eighteen fundamental parameters. The computational steps for evaluating the likelihood function in terms of these parameters are summarized as follows: (1) evaluate $\pi(L)$, $\pi(L^{-1})$, and hence the fourteenth-order symmetric polynomial on the left-hand side of Equation (2.21); (2) factor this polynomial to obtain $R(L)$ in Equation (2.22), use the inverse of $R(L)$ to evaluate the truncated polynomials in (2.23), and thereby obtain the basic G polynomials in the wage equation (2.24); (3) evaluate $(I - C_0)(I - C_0 - C_1)^{-1}$ in Equation (2.26); and (4) compute a time series of vector u_t corresponding to these parameters using Equation (2.26), and from these, compute log-likelihood function (2.28). These function evaluations are then used for computing gradients during the numerical iterations and for computing numerical second-order derivatives for estimating the variance-covariance matrix of the estimated coefficients.

Estimation Results

The estimates of the structural and stochastic shock parameters are given in Table 2-1, along with the ratio of these coefficients to their standard errors as computed from the inverse of the second derivative matrix of the likelihood function. All the structural coefficients have signs and magnitudes that are reasonable. The elasticity of real output with respect to real money balances α_1 corresponds to an income elasticity of money demand of about two-thirds. The estimated elasticity of unemployment with respect to the output gap is .4, which corresponds to an Okun's law multiplier of 2.5. The responsiveness of contract wages to excess demand h is .11, but it is only marginally significantly different from zero. The policy-evaluation procedures, which we report in Section 2.5, are very dependent on h.

The policy parameters g_1, g_2, and g_3 indicate that monetary policy was significantly accommodative during the sample period. The sum of g_1 and g_2, which represents the combined accommodation to wage and price shocks, is .53 with a standard error of .18. (The estimated covariance between the estimates of g_1 and g_2 is $-.019$.) However, the individual accommodation coefficients suggest that it is important to distinguish wages from prices when estimating policy functions. According to these estimates, policy is almost fully accommodative to wages, but it is not at all accommodative to prices.

TABLE 2-1 Maximum-Likelihood Estimates of the Structural Parameters

Parameter	*Estimate*	*Asymptotic "t-Ratio"*
α_1	1.48	5.5
α_2	.40	11.6
h	.11	1.5
g_1	−.46	3.5
g_2	.99	6.2
g_3	−.11	2.1
δ	2.55	5.4
ρ_{y1}	1.30	14.9
ρ_{y2}	−.50	4.9
ρ_p	.78	3.5
ρ_{m1}	1.37	11.7
ρ_{m2}	−.56	6.2
ρ_e	.66	2.1
θ_{x1}	−.58	4.7
θ_{x2}	.10	1.2
θ_{x3}	.17	1.9
θ_{x4}	.06	.7
θ_{xp}	.69	6.1

Maximum value of the log likelihood: 1284.20.
Sample period: 1961:4–1977:4 (7 observations lost due to lags).
Correlation between actual values and sample period simulations:
y: .972; p: .995; m: .975; e: .977; w: .991.

In fact, price shocks seem to generate a restrictive monetary policy, after taking account of the accommodation to wages. Whether this is optimal or not depends on how price shocks enter into the inflationary dynamics, through both expectations and contract effects. Finally, parameter g_3 indicates a countercyclical reaction of monetary policy. When the economy is expected to move below full employment, monetary policy becomes more stimulative.

The estimate of δ, which constrains the contract distribution weights, is most easily interpreted in terms of the π-weights, which can be computed from δ by using the formula $\pi_j/\pi_o = K_1 \exp(-j^2/\delta)$. From the π-weights, the distribution of contract lengths can be computed using Equations (2.12) and (2.13). This distribution is given in Table 2-2. According to these estimates, contract lengths in the three- to four-quarter range appear to predominate. This corresponds to the general view that most implicit contracts are about one year in length.

The parameters of the stochastic processes that describe the stochastic shocks are generally very significant with the exception of the last three unconstrained parameters of the wage shock. The serial correlation matrices presented in Table 2-3 suggest that some serial correlation remains in the estimated residual vectors. The cross-serial correlation parameter between

Table 2-2 Estimated Distribution of Workers by Contract Length

Contract Length in Quarters	*Fraction of Workers in Quarter (f_{jt})*	*Fraction of Workers in Labor Force (a_{jt})*
1	.074	.271
2	.190	.251
3	.234	.199
4	.208	.136
5	.146	.079
6	.084	.039
7	.040	.017
8	.023	.006

price shocks and wage shocks is very significant and indicates that past price shocks do feed back into the wage-formation process.

The implied ARMA coefficients for the model are given in Table 2-4, where the constraints we have imposed are evident. Note that these coefficients represent the simultaneous form of the model. The reduced form would be obtained by multiplying through by A_0^{-1} and would have fewer

Table 2-3 First- through Fourth-Order Serial Correlation Matrices

$$\Gamma_s = \hat{E}(\hat{u}_t, \hat{u}_{t-s})$$

$$\Gamma_1 = \begin{bmatrix} .13 & -.20 & .11 & .01 & .13 \\ -.11 & .33 & -.07 & -.14 & -.47 \\ .23 & -.06 & .19 & .19 & .05 \\ .02 & -.02 & -.05 & .15 & .18 \\ -.01 & .10 & .13 & .09 & .29 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} .08 & -.12 & -.22 & -.21 & .04 \\ -.14 & .24 & .17 & -.06 & -.11 \\ -.10 & -.22 & .10 & -.22 & .03 \\ .34 & -.11 & -.09 & .10 & .09 \\ .15 & -.06 & .16 & .07 & .11 \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} .28 & -.19 & .03 & .00 & -.04 \\ .16 & .31 & .09 & -.04 & .12 \\ .02 & -.10 & .17 & -.15 & .13 \\ .47 & -.18 & .16 & .22 & .10 \\ .19 & -.21 & .18 & .07 & .06 \end{bmatrix}$$

$$\Gamma_4 = \begin{bmatrix} .12 & .04 & -.10 & .00 & -.07 \\ -.19 & .41 & .17 & -.02 & .17 \\ .27 & -.20 & .19 & -.00 & .18 \\ .18 & -.10 & -.08 & -.01 & .06 \\ .25 & .01 & .27 & .13 & .17 \end{bmatrix}$$

Note: The order of the elements of these matrices is y, p, m, e, w.

Table 2-4 Constrained Simultaneous VARMA Model

$$A_0 y_t = A(L)y_t + B(L)u_t$$

	Lag										
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
$A(2,5)$	.359	.230	.131	.066	.028	.010	.003				
$A(3,5)$	.216	.138	.079	.034	.017	.006	.002				
$A(5,5)$	.359	.230	.131	.066	.028	.010	.003				
$B(1,1)$	1.230	1.189	.897	.572							
$B(2,1)$	.016	.024	.026	.022	.015	.009	.005	.002	.001	.000	.000
$B(2,2)$	1.108	1.123	1.041	.883	.362	.227	.125	.061	.024	.007	.002
$B(2,3)$	.025	.040	.043	.036	.025	.015	.008	.004	.001	.000	.000
$B(2,4)$	.017	.026	.027	.022	.015	.009	.004	.002	.001	.000	.000
$B(2,5)$	.376	.445	.535	.470	.337	.219	.127	.057	.031	.010	.002
$B(3,1)$	−.115	−.099	−.007	−.004	−.009	.005	.003	.001	.000	.000	.000
$B(3,2)$	.001	.002	.218	.212	.217	.136	.075	.037	.014	.005	.001
$B(3,3)$	1.188	1.142	.891	.574	.015	.009	.005	.002	.001	.000	.000
$B(3,4)$	.010	.016	.016	.013	.009	.005	.003	.001	.000	.000	.000
$B(3,5)$	.226	.267	.321	.282	.202	.131	.076	.034	.019	.006	.001
$B(4,4)$	.656	.430	.282	.185							
$B(5,1)$	.016	.024	.026	.022	.015	.009	.005	.002	.001	.000	.000
$B(5,2)$	.326	.511	.562	.508	.362	.227	.125	.061	.036	.007	.002
$B(5,3)$	.025	.040	.043	.036	.025	.015	.008	.004	.001	.000	.000
$B(5,4)$	.017	.026	.027	.22	.015	.009	.005	.002	.001	.000	.000
$B(5,5)$	.376	.445	.535	.470	.337	.219	.127	.057	.031	.010	.002

Note: Each column represents a matrix coefficient stacked by rows from the matrix polynomial $A(L)$ or $B(L)$. If a component is not listed, then the coefficients corresponding to that component are constrained to zero for all lags. If there is no entry for a listed component, then that coefficient is constrained to zero.

elements constrained to equal zero. For example, some of the reduced-form dynamics in y_t are due to past movements in m_t and p_t. These would be evident in the reduced form but are only implicit in the simultaneous form of the model.

The constraints that the model and the expectations assumptions put on the wage and price dynamics are evident in the second and fifth rows of matrices A and B given in Table 2-4. The second row corresponds to the price dynamics and the fifth row to the wage dynamics. With one important exception, these dynamics are the same. The exception is that $B(2,2) \neq B(5,2)$. The $B(2,2)$ coefficients are partially determined by the impact of non-wage shocks on the pricing process; that is, the influence of the other components of unit costs, such as productivity shifts. The $B(5,2)$ coefficients reflect the impact of these same price shocks on wages.

Note, however, that both $B(2,2)$ and $B(5,2)$, as well as most of the other elements of matrices A and B depend on the policy parameters g_1, g_2, and g_3. As these coefficients change, the coefficients of A and B will change in a predictable way. It is this impact of the policy parameters on the dynamics of the model that will form the basis of the policy-evaluation procedure.

2.4 The Effect of Policy Changes on the Reduced-Form Coefficients

The estimated structural equations of the model are summarized below:

$$
\begin{aligned}
y_t &= 1.48(m_t - p_t) + u_{yt} + 1.30u_{yt-1} + 1.19u_{yt-2} + .90u_{yt-3} + .57u_{yt-4} \\
p_t &= \hat{w}_t + u_{pt} + .78u_{pt-1} + .63u_{pt-2} + .48u_{pt-3} + .30u_{pt-4} \\
m_t &= g_1\hat{p}_t + g_2\hat{w}_t - .11\hat{y}_t + u_{mt} + 1.37u_{mt-1} + 1.36u_{mt-2} \\
&\quad + 1.22u_{mt-3} + .96u_{mt-4} \\
e_t &= .40y_t + u_{et} + .66u_{et-1} + .47u_{et-2} + .33u_{et-3} + .18u_{et-4} \\
w_t &= .27x_t + .248x_{t-1} + .199x_{t-2} + .137x_{t-3} + .82x_{t-4} + .042x_{t-5} \\
&\quad + .019x_{t-6} + .007x_{t-7} \\
x_t &= .267\hat{w}_t + .248\hat{w}_{t+1} + .199\hat{w}_{t+2} + .137\hat{w}_{t+3} + .082\hat{w}_{t+4} + .042\hat{w}_{t+5} \\
&\quad + .019\hat{w}_{t+6} + .007\hat{w}_{t+7} + .029\hat{e}_t + .027\hat{e}_{t+1} + .022\hat{e}_{t+2} \\
&\quad + .015\hat{e}_{t+3} + .009\hat{e}_{t+4} + .005\hat{e}_{t+5} + .002\hat{e}_{t+6} + .001\hat{e}_{t+7} \\
&\quad + .69u_{pt-1} + .48u_{pt-2} + .35u_{pt-3} + .22u_{pt-4} + u_{xt} \\
&\quad - .58u_{xt-1} + .10u_{xt-2} + .17u_{xt-3} + .06u_{xt-4}.
\end{aligned}
$$

The estimated values of g_1 and g_2 are $-.46$ and .99 respectively. In writing down the model, I do not enter these specific values since I will be concerned with varying these parameters. The policy-evaluation problem concerns how variations in the policy parameter g_1 and g_2 affect the performance of the economy.

As shown in Section 2.2, the model can be reduced to a five-dimensional VARMA model the coefficients of which depend on the policy parameters. The 5×5 matrix A_0 in Equation (2.27) contains structural parameters that are assumed not to depend on the policy parameters. The matrix polynomials $A(L)$ and $B(L)$, which are seventh and eleventh orders respectively, depend explicitly on the policy parameters. The relationship between the reduced form and the policy parameters does not have a closed form but can be evaluated numerically. This is illustrated by comparing Tables 2-4 and 2-5.

Recall that Table 2-4 gives the values of the matrix polynomials $A(L)$ and $B(L)$ when the policy parameters are set to their estimated values along with all the other parameters in the model. These parameter values for $A(L)$ and $B(L)$ are the maximum-likelihood estimates of the ARMA model as constrained by the rational expectations relationships between the equations. As mentioned above, the estimated values of g_1 and g_2 are $-.46$ and .99 respectively. When these parameter values are changed, the parameters of $A(L)$ and $B(L)$ will also change. Recall that the Lucas critique for conventional econometric policy evaluation, as explained in Chapter 1, is that parameters of models change when policy changes. The calculations in Table 2-5 show how we have incorporated the Lucas critique into the policy-evaluation procedure. For example, when g_1 is increased from $-.46$ to 0, representing a more accommodative policy, the parameters of $A(L)$ and $B(L)$ shift from those given in Table 2-4 to those given in Table 2-5. Note that most of the 172 parameters in Tables 2-4 and 2-5 change as a result of the shift in the single parameter g_1. The only parameters that do not change are the eight exogenous serial correlation parameters in the output and unemployment equations ($B[1, 1]$ and $B[4, 4]$). This strong interaction between policy parameters and econometric equations is due to the rational expectations assumption.

Considering first the autoregressive parameters in the wage equation $A(5, 5)$, the effect of the more accommodative policy is to raise uniformly these autoregressive weights. The moving average weights in the wage equation are also increased. As one would expect, moving to an easier monetary policy raises the coefficients of the lagged dependent variables in the wage equation and thereby increases the persistence of wages. Because of the markup relationship between wages and prices, the effect of a more accommodative policy on the price dynamics is similar to that of wages.

The $B(3, 2)$ parameters measure the impact of price changes on the money supply. These enter not only directly through the forecast of prices but also through the wage forecast in the money-supply rule. The increase in the parameter g_1 has the effect of increasing the response of the money supply to price movements as reflected by the $B(3, 2)$ parameters, which are much higher in Table 2-5 than in Table 2-4.

Table 2-5 ARMA Parameters with More Accommodative Policy ($g_1 = 0$; $g_2 = .99$)

	Lag										
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
$A(2,5)$	.429	.270	.152	.075	.032	.011	.003				
$A(3,5)$	.425	.268	.150	.074	.032	.011	.003				
$A(5,5)$	.429	.270	.151	.075	.032	.011	.003				
$B(1,1)$	1.300	1.189	.897	.572							
$B(2,1)$	.018	.028	.029	.025	.017	.010	.005	.003	.001	.000	.000
$B(2,2)$	1.170	1.215	1.135	.961	.415	.259	.142	.069	.026	.008	.002
$B(2,3)$	.029	.046	.048	.040	.028	.017	.009	.004	.002	.000	.000
$B(2,4)$	.020	.029	.030	.025	.017	.010	.005	.002	.001	.000	.000
$B(2,5)$	.375	.484	.594	.526	.379	.246	.142	.065	.034	.010	.002
$B(3,1)$	−.107	−.086	−.057	−.030	.017	.010	.005	.003	.001	.000	.000
$B(3,2)$	.495	.684	.718	.634	.411	.257	.141	.068	.026	.008	.002
$B(3,3)$	1.202	1.163	.914	.592	.028	.017	.008	.004	.001	.000	.000
$B(3,4)$	.020	.029	.030	.025	.017	.010	.005	.002	.001	.000	.000
$B(3,5)$	.372	.480	.588	.520	.375	.244	.141	.064	.034	.011	.002
$B(4,4)$	.656	.430	.282	.185							
$B(5,1)$	.018	.028	.029	.025	.017	.010	.005	.003	.001	.000	.000
$B(5,2)$	.388	.603	.656	.587	.415	.259	.142	.069	.026	.008	.002
$B(5,3)$	.029	.046	.048	.040	.028	.017	.009	.004	.002	.000	.000
$B(5,4)$	.020	.029	.030	.024	.017	.010	.005	.002	.001	.000	.000
$B(5,5)$	.375	.484	.594	.526	.379	.246	.142	.065	.034	.011	.002

2.5 Design of Policy Rules

In the remainder of this chapter, I consider two modes of policy analysis: design of a policy rule and transition to a new policy rule. In this section, I consider the design question and ask how the economy would behave over an extended period of time if different policy rules were in place. The distribution of the endogenous variables indicates how the economy would operate if subject to random shocks with the same variance-covariance structure as those observed during the sample period. Since I have normalized the endogenous variables to have zero means, I focus on the steady-state variance-covariance matrix of the distribution of the endogenous variables in z_t.

Steady-State Covariance Matrix of the Endogenous Variables

From Equation (2.27) we can obtain

$$\begin{aligned} z_t &= A_0^{-1} A(L) z_t + A_0^{-1} B(L) u_t \\ &= [I - A_0^{-1} A(L)]^{-1} A_0^{-1} B(L) u_t \\ &= \sum_{i=0}^{\infty} \theta_i u_{t-i}, \end{aligned} \tag{2.35}$$

where the matrix series θ_i is a function of A_0, $A(L)$, and $B(L)$ and hence is a function of g_1 and g_2. The steady-state covariance matrix of the vector of endogenous variables z_t is therefore equal to

$$V(g_1, g_2) = \sum_{i=0}^{\infty} \theta_i \Omega \theta_i', \tag{2.36}$$

where Ω is the variance-covariance matrix of the serially uncorrelated shocks u_t. If the system (2.27) is stable, then (2.36) is a convergent series and hence, can be evaluated to within any desired level of accuracy.

We will focus primarily on the diagonal elements of V, the variances of the steady-state distribution of each endogenous variable. Table 2-6 shows these diagonal elements in percentage standard-deviation units. Recall that each variable is measured as a deviation from secular trend; for example, when $g_1 = 0$ and $g_2 = .99$, the case illustrated in Table 2-5, the standard deviation of output around trend is 1.5 percent.

Most of the variation in the policy parameters in Table 2-6 is confined to g_2, with g_1 held to zero. The reason for this is that the *sum* of g_1 and g_2 is quantitatively more important than the individual values. This is illustrated by comparing the results when $g_1 = 0$ and $g_2 = .50$ with $g_1 = -.46$ and $g_2 = .99$. The steady-state distribution is the same in both cases, and the sum $g_1 + g_2$ is close to .5 in both cases. The same results follow from comparing the last two rows in Table 2-6. Evidently, the total degree of accommodation

Table 2-6 Effect of Policy Parameters on the Behavior of the Endogenous Variables

g_1	g_2	σ_y	σ_p	σ_m	σ_u	σ_w
.0	.99*	1.5	8.8	8.8	0.5	8.7
.0	.95	1.5	6.0	5.8	0.5	5.8
.0	.90	1.6	5.0	4.6	0.5	4.8
.0	.85	1.7	4.5	4.0	0.6	4.3
.0	.80	1.7	4.1	3.5	0.6	3.9
.0	.75	1.8	3.9	3.2	0.6	3.7
.0	.70	1.9	3.7	3.0	0.7	3.5
.0	.65	2.0	3.6	2.7	0.7	3.3
.0	.60	2.1	3.4	2.5	0.8	3.2
.0	.55	2.2	3.3	2.4	0.8	3.1
.0	.50	2.3	3.2	2.2	0.8	3.0
−.46*	.99*	2.3	3.2	2.2	0.8	3.0
.0	.45	2.4	3.1	2.1	0.9	2.9
.0	.40	2.5	3.1	1.9	0.9	2.8
.0	.35	2.5	3.0	1.8	0.9	2.7
.0	.30	2.6	2.9	1.7	1.0	2.6
.0	.25	2.7	2.9	1.7	1.0	2.6
.0	.20	2.8	2.8	1.6	1.0	2.5
.0	.15	2.9	2.8	1.5	1.1	2.5
.0	.10	3.0	2.7	1.5	1.1	2.4
.0	.05	3.0	2.7	1.4	1.1	2.4
.0	.0	3.1	2.7	1.4	1.2	2.3
−.46*	.50	3.1	2.7	1.4	1.2	2.4

Note: The policy parameters g_1 and g_2 are the elasticities of the money supply with respect to prices and wages respectively, as given by the policy rule. The σ-parameters are the standard deviations (in percent) evaluated at the equilibrium distribution as a function of these policy parameters. The asterisk represents the estimated value of the policy parameter during the sample period.

is more significant in this model than the differential accommodation between prices and wages. This result reflects both the markup assumption we have used for determining prices and the nature of the feedback of prices into the wage equation.

In any case, because only the sum of g_1 and g_2 is quantitatively important, most of the policy comparisons in Table 2-6 consider only variations in g_2. Alternative policies range from almost full accommodation in the top row of Table 2-6 to no accommodation in the last two rows. As policy moves in a less accommodative direction, the variability of real variables, output, and unemployment increases, whereas the variability of the nominal variables, prices, wages, and of the money supply, decreases. The estimated policy ($g_1 = -.46$ and $g_2 = .99$) is halfway between these two extremes.

Focusing on σ_y and σ_p, it is clear that small changes in policy in either direction from the estimated policy, lead to point-for-point changes in output variability and price variability. That is, when the standard deviation of output increases by .1 percent, the standard deviation of prices falls by .1 percent. This may seem paradoxical in the sense that wages and prices are "sticky" in this model, so that changes in money should result in more of a change in output than in prices. The paradox is resolved by noting that the standard deviation of these variables measures their behavior on average over a long period of time. In the short run, wages and prices may be rigid, but in the long run, they adjust. Measures of economic performance, such as the standard deviation in steady-state distribution, combine these two features.

Note that this point-for-point trade-off changes as the policy parameters move away from the estimated policy. At the extreme of a very accommodative policy, reductions in output variability are accompanied by very large increases in price variability. Similarly, at the extreme of nonaccommodation, more price stability is very costly in terms of increased output variability.

Comparison of Policy Rules with Historical Shocks

Comparative policy analysis in econometric models has been traditionally achieved by simulating alternative paths for the policy variables in the reduced form of the model over some historical period. The steady-state covariance matrix presented is conceptually different from such comparisons since it is not restricted to a particular episode. In effect, the performance of the economy is evaluated over some arbitrary period with the shocks generated by the distribution of the residuals in each equation. Here I address the policy-evaluation problem by asking how the economy would have performed in the mid-1970s if alternative policy *rules* had been in operation at the start of the simulation period and had been maintained throughout.

Technically, this is done by simulating the model with the actual estimated shocks in each equation in each time period, but with different policy rules determining the response of the money supply to these shocks. We consider two alternatives to the estimated policy: a more accommodative policy with $g_1 = 0$ and $g_2 = .99$ and a less accommodative policy with $g_1 = 0$ and $g_2 = 0$. These alternative policies correspond to the first and last rows of Table 2-6.

The results of this comparison are presented in Tables 2-7 and 2-8. The inflation effects of the different policies are shown in Table 2-7, while the output effects are shown in Table 2-8. As before, output is measured as a deviation from a secular trend. For each of the policies, a trade-off between higher inflation and higher output levels is evident. (Such a trade-off does not exist in the long run, however. It appears here because only a single episode is considered.) For the estimated policy, the rate of inflation averaged 7.1 percent over the 4 years ending in 1977:4, while output averaged 3.1 percent below normal. The more accommodative policy cuts this output loss to near zero but results in a much higher rate of inflation (8.8 percent), which is accelerating rapidly at the end of the simulation period. The less

TABLE 2-7 Inflation Effects of Alternative Policy Rules, 1973:2–1977:4 (quarterly percent change in the GNP deflator, seasonally adjusted annual rate)

	Estimated Policy	*More Accommodative Policy*	*Less Accommodative Policy*
73:2	7.0	6.0	6.4
73:3	7.4	6.7	6.6
73:4	8.6	8.8	8.4
74:1	8.7	7.9	7.2
74:2	9.8	10.8	10.0
74:3	12.2	11.6	10.0
74:4	13.5	12.8	10.8
75:1	8.5	11.6	8.8
75:2	4.3	7.6	4.3
75:3	7.1	8.8	5.9
75:4	6.6	7.6	4.8
76:1	3.9	5.2	2.9
76:2	4.7	6.4	3.6
76:3	4.5	6.4	3.6
76:4	5.7	7.4	4.4
77:1	6.0	7.8	4.8
77:2	7.7	11.6	6.4
77:3	5.1	7.6	3.9
77:4	5.5	10.4	6.8
4 quarters ending:			
74:4	11.1	10.7	9.5
75:4	6.6	8.9	6.0
76:4	4.7	6.4	3.6
77:4	6.1	9.3	5.4
4 years ending:			
77:4	7.1	8.8	6.1

Note:
For the estimated policy, $\hat{g}_1 = -.46$ and $\hat{g}_2 = .99$.
For the more accommodative policy, $g_1 = 0$ and $g_2 = .99$.
For the less accommodative policy, $g_1 = 0$ and $g_2 = 0$.

accommodative policy increases the output loss and has a corresponding reduction in the rate of inflation.

The inflation reduction that is associated with the less accommodative policy as compared with the actual policy, is somewhat more than implied by many econometric models without rational expectations or without an explicit model of contracts. A consensus in the late 1970s was that "the cost

Table 2-8 Output Effects of Alternative Policy Rules, 1973:2–1977:4 (percent deviation of real GNP from secular trend)

	Estimated Policy	*More Accommodative Policy*	*Less Accommodative Policy*
73:2	3.9	2.1	4.0
73:3	3.5	2.0	3.3
73:4	3.2	1.9	2.7
74:1	1.3	0.6	1.0
74:2	0.0	−0.1	−0.4
74:3	−1.5	−0.3	−2.0
74:4	−3.7	−1.7	−4.8
75:1	−6.9	−3.9	−8.7
75:2	−6.3	−2.5	−8.6
75:3	−4.6	−1.4	−7.2
75:4	−4.8	−1.3	−6.7
76:1	−3.4	0.1	−4.8
72:2	−3.3	0.7	−4.8
76:3	−3.5	0.8	−5.3
76:4	−3.8	0.2	−6.2
77:1	−2.9	1.3	−5.4
77:2	−2.3	2.4	−5.2
77:3	−1.7	3.4	−4.9
77:4	−2.5	2.7	−5.8
Average for 4 years ending:			
1977:4	−3.1	0.1	−5.1

Note: See Table 2-7 for the specific parameter values associated with each policy.

of a *1-point* reduction in the basic inflation rate is *10 percent* of a year's GNP" (Okun, 1978). According to the columns of Tables 2-7 and 2-8, if real output averaged 2 percent below the actual performance, inflation would have been about 1 percent lower on average.

Response to Individual Wage Shocks

Insight into how the economic system behaves can be gained by looking at the effects of isolated wage shocks. I focus on temporary unanticipated wage shocks and examine only the output and price responses. The simulation of a single wage shock is performed just as a temporary unanticipated shock, as explained in Chapter 1, Section 1.2.

Table 2-9 shows the response of the system to a temporary wage shock under the same three alternative policy rules examined above. Three general properties of the model and of the policies are worth emphasizing.

Table 2-9 Effect of an Unanticipated Temporary Wage Shock (shock equals 10 percent in initial quarter, zero thereafter)

Quarter	*Estimated Policy*		*More Accommodative Policy*		*Less Accommodative Policy*	
	y	*p*	*y*	*p*	*y*	*p*
0						
1	−4.2	7.0	−1.1	7.7	−8.5	6.7
2	−5.3	9.0	−1.5	10.5	−10.5	8.3
3	−6.6	11.1	−1.9	13.7	−12.8	10.1
4	−7.2	12.1	−2.2	15.6	−13.5	10.6
5	−7.1	12.0	−2.3	16.5	−13.0	10.2
6	−6.9	11.6	−2.4	16.8	−12.1	9.5
7	−6.4	10.8	−2.4	16.9	−10.9	8.5
8	−5.9	9.9	−2.3	16.7	−9.5	7.5
9	−5.4	9.1	−2.3	16.5	−8.5	6.6
10	−4.9	8.3	−2.2	16.3	−7.4	5.8
11	−4.5	7.6	−2.2	16.0	−6.5	5.1
12	−4.1	6.9	−2.2	15.8	−5.7	4.4
13	−3.8	6.3	−2.2	15.6	−5.0	3.9
14	−3.4	5.8	−2.1	15.4	−4.3	3.4
15	−3.1	5.3	−2.1	15.2	−3.8	3.0
16	−2.9	4.9	−2.1	14.9	−3.3	2.6
17	−2.6	4.4	−2.0	14.7	−2.9	2.3
18	−2.4	4.1	−2.0	14.5	−2.6	2.0
19	−2.2	3.7	−2.0	14.3	−2.3	1.8
20	−2.0	3.4	−2.0	14.1	−2.0	1.6
21	−1.8	3.1	−1.9	13.9	−1.7	1.4
22	−1.7	2.8	−1.9	13.8	−1.5	1.2
23	−1.5	2.6	−1.9	13.6	−1.3	1.0
24	−1.4	2.4	−1.8	13.4	−1.2	0.9
25	−1.3	2.2	−1.8	13.2	−1.0	0.8

Note: See Table 2-7 for the specific parameter values associated with each policy. Note that *y* is the deviation of real GNP from the baseline and *p* is the deviation of the log of the GNP deflator from the baseline.

First, the gradual impact of a wage shock on both prices and output is evident in Table 2-9. The peak effect of the shock occurs after four quarters for the estimated policy and the less accommodative policy and after seven quarters for the more accommodative policy. This gradual impact is due to the staggered wage contracts: it takes several periods before a shock passes through the several levels of contracts.

Second, note that the persistence of the wage shock depends very heavily on which policy is being used. For the more accommodative policy, the wage shock is still above the peak of the other two policies after twenty-five quarters and is diminishing very slowly. The persistence of the price behavior is mirrored by the output behavior. Although the depth of the downturn is much lower for both the estimated and the less accommodative policies, these downturns do not last as long. Moreover, after twenty quarters, the other two policies "overtake" the less accommodative policy and result in higher output levels.

Finally, Table 2-9 indicates very clearly the difference between the long-run and the short-run effects of policy. These were mentioned as an explanation for the point-for-point trade-off between the standard deviations of output and prices. In the short run, a comparison of the three policies shows how the temporary rigidity of wages implies that output takes the major effect of a tighter monetary policy. The main difference between the three policies in the first several quarters shows up in output rather than in prices. However, in the longer run, most of the difference is found in price behavior rather than in output behavior. Comparing rows 1 and 25 of Table 2-9 shows this effect most dramatically.

2.6 Transition to a New Policy Rule: Disinflation

The policy evaluation presented in the previous section considers how different monetary policy rules influence the deviation of actual prices and output from a trend. In this section, I consider the problem of changing to a less inflationary policy rule and examine the output effects that are associated with such a change.

Consider a situation in which the rate of inflation is viewed as too high, and the objective of monetary policy is to bring this inflation rate to a lower level, to disinflate the economy. Clearly, disinflation requires a reduction in the rate of monetary growth. The important question is how fast this reduction in money growth should be. The "gradualist" proposal is that the reduction in money growth should be slow. One rationale for the gradualist approach is that outstanding contracts, such as the contracts described in the model discussed here, will translate a sudden reduction in money growth into a large loss in output and employment. A gradual reduction in money growth will give some time for contracts to adjust.

The output effects associated with an announced program of monetary disinflation, either gradual or sudden, can be evaluated using the model of this chapter by changing the money-supply rule to the following simple form:

$$m_t = [(1 - L^2)(1 - kL)]^{-1}(1 - k)\, u_{mt}, \tag{2.37}$$

where the disturbance term u_{mt} is serially uncorrelated. An announced monetary disinflation (unanticipated before the announcement date) can be characterized by a *particular realization* of the disturbance process u_{mt}. For example, if u_{mt} equals $-.0025$ in quarter $t = 1$ and zero thereafter, then a permanent 1-percent (annual rate) reduction in money growth begins

TABLE 2-10 Inflation and Output Effects of a 1-Percentage Point Reduction in Money Growth (growth at annual rates)

	Immediate Reduction in Money Growth		
Quarter	*Money Growth Rate*	*Inflation Rate*	*GNP Gap (percent)*
0	10.00	10.00	.00
1	9.00	10.00	.37
2	9.00	9.32	.50
3	9.00	9.07	.52
4	9.00	8.93	.50
5	9.00	8.85	.44
6	9.00	8.85	.38
7	9.00	8.86	.33
8	9.00	8.87	.29
9	9.00	8.90	.25
10	9.00	8.91	.21
11	9.00	8.92	.18
12	9.00	8.93	.15
	Gradual Reduction in Money Growth		
Quarter	*Money Growth Rate*	*Inflation Rate*	*GNP Gap (percent)*
0	10.00	10.00	.00
1	9.50	10.00	.19
2	9.25	9.39	.24
3	9.12	9.16	.25
4	9.06	9.03	.24
5	9.03	8.95	.21
6	9.02	8.93	.18
7	9.01	8.94	.15
8	9.01	8.94	.13
9	9.00	8.95	.11
10	9.00	8.96	.09
11	9.00	8.97	.08
12	9.00	8.97	.07

in quarter $t = 1$ and is perfectly anticipated starting at that time. If $k = 0$, then the 1-percent reduction in money growth occurs entirely in the first period. If k is greater than zero, then the reduction is gradual; more specifically, it is phased in geometrically. It may be useful to recall the distinction between the effects of one-time policy shocks and the effects of policy rules emphasized in Section 1.2 of Chapter 1. Here we are considering a one-time shock.

Table 2-10 shows the effects of such a monetary disinflation for values of k equal to 0 and .5. It is assumed that the previous rate of inflation was

10 percent and that all other shocks to the model are set to zero during the disinflation. Hence, the important question about future accommodation is ignored. Suppose that the goal is to reduce the rate of inflation by one percentage point.

When the disinflation is immediate, the output loss is larger than with the gradualist policy. Although the rate of inflation does not reach the new target as quickly under a gradualist path, there is not as much overshoot before the inflation rate settles at the new equilibrium. Overall, the advantages of a gradualist policy as compared to a more sudden change in money growth are clearly illustrated in this comparison. The total output loss associated with the sudden 1-percent disinflation is about 4 1/2 percent of GNP. The gradualist policy cuts this loss in half.

2.7 Model Validation with Policy Analysis

Much has been made of the importance of validating econometric models by evaluating their forecasting accuracy. One looks at how well the model forecasts beyond the sample period over which the model has been estimated. Forecasting is an especially good model-validation tool if the forecasting is done for a period after the date on which the econometrician actually estimates the model. Then there is no way that the econometrician could have peeked at the data to fit the model so as to generate good forecasts.

Although rarely done, policy evaluation is also a good model-validation procedure, especially for models whose purpose is policy analysis rather than pure forecasting. Is the policy analysis of the model more accurate in retrospect than other analyses? If so this would lend support for the model and the approach.

It turns out that such a validation is feasible with the model presented in this chapter. As described above, the simulations of output loss under alternative disinflation paths were performed on the basis of the model estimated with data from the 1960s and 1970s before the disinflation of the early 1980s. The calculations reported in the previous two sections were made before the disinflations of the 1980s and were distributed in unpublished working papers (See Reference Notes at the end of the chapter).

Were the estimates of output loss accurate? How do the estimates compare with the actual experience of disinflation in the 1980s? Are the estimates more accurate than other policy analyses?

The calculations from the rational expectations model suggest that the output loss associated with a sudden disinflation would be around 4 1/2 percent for each percentage-point decline in the inflation rate (see the discussion at the end of Section 2.6). A more gradual disinflation would lower this number. As mentioned, conventional estimates in the late 1970s suggested that the loss would be much greater than this: around 10 percent of a year's GNP for 1-percent inflation reduction is the average estimate summarized by Okun (1978). These conventional estimates come from traditional models without rational expectations. Which output loss ratio is more accurate?

Work backward from the unemployment rate: during the seven years from 1980 through 1986 the difference between the unemployment rate and a 6.5-percent natural unemployment rate summed to 10.2 percent. With an Okun's law coefficient of 2.5, this implies a cumulative output loss of 26 percent. The underlying inflation rate fell from about 10 percent to about 4 percent, or by about 6 percentage points during this period. (The decline was larger than 6 percentage points for the underlying consumer price index (CPI) inflation rate and slightly less for the GNP deflator.) Dividing the output loss (26 percent) by the inflation decline (6 percent) gives an actual output loss ratio of about 4.3 percent. This number is remarkably close to the 4 1/2 number calculated in 1980 with the model presented in this chapter and reported at the end of Section 2.6. It is certainly much closer than the conventional estimates at the time.

Such a comparison of course depends on an assumption that other factors did not affect the relationship between inflation and unemployment. It also depends on an assumption about the natural rate of unemployment and the underlying inflation rate. However, for these calculations, such factors seem unlikely to change the estimates by much. The estimates provide considerable validation of the model and of the approach.

2.8 Policy Summary, Retrospect, and Prospect

The purpose of this chapter has been to illustrate the general econometric policy-evaluation approach by comparing alternative monetary policy rules and disinflation paths in a small econometric business cycle model of the United States. Although many types of policy problems could be addressed within this framework, the analysis has focused primarily on how different degrees of monetary accommodation affect the behavior of real output versus inflation. Since the model contains both rational expectations and an explicit process for the determination of wage contracts, it is especially suited for this type of comparison.

The implications of these simulations can be summarized as follows: (1) changes in the monetary policy rule trace out a trade-off between the variability of output and prices; (2) although more accommodative monetary policy rules do reduce the depth of recessions, they tend to increase their length (the model suggests that five years after a price shock sets off a recession, a less accommodative monetary policy would lead to output levels that are higher than more accommodative policies); and (3) the trade-off between output and inflation, which is implicit in this model, is considerably more favorable (in the sense that smaller output reductions are associated with a given reduction in inflation) than traditional econometric models without rational expectations or explicit wage contracts would have suggested. A comparison of *ex ante* calculations of the costs of disinflation with this model and with what actually happened in the 1980s provides validation for the model.

In commenting on this model in the early 1980s, Robert Lucas said, “We have come a very long way toward restoring seriousness to our discussions of macroeconomic policy, perhaps as far as we can go on the basis of the theory and the evidence we have so far processed” (1981, p. 565). The “very long way” was in comparison to conventional models and reflects the use of rational expectations and the focus on policy rules. The purpose of the rest of this book is to build on this model in ways that were infeasible ten years ago. Development in both economic theory and econometric theory, as well as evidence from two recessions, a major disinflation and a long expansion, have provided a basis for considerable progress, as I think the following chapters show.

Reference Notes

The economywide model of the United States introduced in Section 2.1 is drawn from two of my unpublished papers: “An Econometric Business Cycle Model with Rational Expectations: Some Estimation Results” and “An Econometric Business Cycle Model with Rational Expectations: Policy Evaluation Results,” which were circulated and discussed in seminars in the early 1980s. I initially viewed this single-country linear model as an intermediate step between my 1979 *Econometrica* paper and a more general, but more complicated, multicountry model such as the one described in Chapter 3. Because the model is “intermediate,” it is a good expositional device for econometric policy-evaluation procedures.

The Davidon-Fletcher-Powell method for maximizing the likelihood function in Section 2.2 was originally introduced in Davidon (1959) and Fletcher and Powell (1963). Fair (1984) provides a good exposition of how this and other methods are used in maximum-likelihood estimation. Wallis (1980) discusses estimation and identification in rational expectations models like the one estimated here. Dawn Rehm’s (1982) Columbia University Ph.D. dissertation estimated open economy models for the United States and Germany with more complicated structures than the model in this chapter, although using similar solution and estimation methods.

A derivation and intuitive explanation of the trade-off between the variance of real output and the variance of the price level in Section 2.5 is provided in Taylor (1980).