THE IMPLICATIONS OF UNCERTAINTY FOR MONETARY POLICY

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Abstract

In this paper we use a simple model of the Australian economy to empirically examine the consequences of parameter uncertainty for optimal monetary policy. Optimal policy responses are derived for a monetary authority that targets inflation and output stability. Parameter uncertainty is characterised by the estimated distribution of the model coefficient estimates. Learning is ruled out, so the monetary authority can commit to its *ex ante* policy response. We find that taking account of parameter uncertainty can recommend more, rather than less, activist use of the policy instrument. While we acknowledge that this finding is specific to the model specification, parameter estimates and the shocks analysed, the result does stand in contrast to the widely held belief that the generic implication of parameter uncertainty is more conservative policy.

JEL Classification Numbers: E52, E58

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1. Introduction

Monetary authorities aim to achieve low and stable inflation while keeping output at capacity. To achieve these goals they manipulate the policy instrument which has an effect on economic activity and prices through one or more transmission mechanisms. Monetary authorities face many difficulties in achieving these goals. The current state of the economy, for example, is not known with certainty. Moreover, the responses of the economy to demand and supply shocks are difficult to quantify and new shocks are arriving all the time. As if these problems are not enough, the transmission channels from the policy instrument to the objectives are complex and imprecisely estimated.

Economic models are useful tools for helping to deal with these uncertainties. By abstracting from less important uncertainties, models provide a framework within which the workings of the economy can be quantified. In doing so, models generally reduce the complexity of the policy decision-making process and go some way towards helping monetary authorities achieve their goals. However, to the extent that models are only an approximation to the 'true' economy, there will always be uncertainty about the correct structure and parameters of an economic model.

Blinder (1995), commenting in his capacity as a central banker, observed that model uncertainty can have important implications for policy. In particular, uncertainty about the model may make monetary authorities more conservative in the sense that they determine the appropriate policy response ignoring uncertainty, 'and then do less'. This conservative approach to policy was first formalised by Brainard (1967). Although Blinder views the Brainard conservatism principle 'as extremely wise', he admits that the result is not robust. For practical purposes, this recommendation leaves two questions unanswered. First, how much should policy be adjusted to account for model uncertainty? Second, is conservatism always the appropriate response?

This paper addresses both of these questions by generalising the Brainard model to a multi-period horizon and a multivariate model. A small data-consistent model of the Australian economy is used to illustrate the effect of parameter uncertainty on policy responses. Contrary to Brainard's conservatism result, we show that parameter uncertainty can actually induce *greater policy activism* following most types of shocks. We argue that this increased activism is primarily a consequence of uncertainty about the persistence of shocks to the economy. This type of uncertainty cannot be incorporated into the static model of Brainard.

The remainder of the paper is structured as follows. In Section 2, we discuss the various sources of forecasting error which lie behind model uncertainty. Section 3 summarises the specification of a small macroeconomic model used in the remainder of the paper. Section 4 shows how sensitive policy responses are to changes in the parameter values when the policy-maker ignores this parameter uncertainty. Section 5 demonstrates how parameter uncertainty can be accommodated in the solution to a monetary authority's optimal policy problem and Section 6 illustrates the difference between naive policy, that ignores parameter uncertainty, and policy that explicitly takes parameter uncertainty into account. Section 7 concludes and summarises the practical implications for monetary policy.

2. Sources of Forecast Uncertainty

Clements and Hendry (1994) provide a taxonomy of forecast error sources for an economic system that can be characterised as a multivariate, linear stochastic process. This system can generally be represented as a vector autoregressive system of linear equations. Furthermore, most models of interest to policy-makers can be characterised by a set of trends that are common to two or more variables describing the economy. In these cases, the economic models can be written as vector error-correction models.

For these models, forecast errors can come from five distinct sources:

1. structural shifts;
2. model misspecification;
3. additive shocks affecting endogenous variables;
4. mismeasurement of the economy (data errors); and
5. parameter estimation error.

The first source of forecasting error arises from changes in the economic system during the forecast period. The second source of forecasting error may arise if the model specification does not match the actual economy. This may arise, for example, if the long-run relationships or dynamics have been incorrectly specified. Forecasting errors will also arise when unanticipated shocks affect the economic system. These shocks accumulate, increasing uncertainty with the length of the forecast horizon. If the initial state of the economy is mismeasured then this will cause persistent forecast errors. Finally, forecast errors may also arise because finite-sample parameter estimates are random variables, subject to sampling error.

By definition, without these sources of forecast error, there would be no uncertainty attached to the forecasts generated by a particular system of equations. Recent research by Clements and Hendry (1993, 1994 and 1996) explores the relative importance of each of these sources of forecasting error. They find that structural shifts in the data generating process or model misspecifications, resulting in intercept shifts, are the most persistent sources of forecasting error in macroeconomic models.

Rather than comparing the relative importance of each of these sources of uncertainty for forecasting, this paper makes a contribution toward understanding the implications of parameter uncertainty for monetary policy decision-making. In the context of our economic model, we solve for ‘optimal policy’ explicitly taking into account uncertainty about the parameter estimates. Comparing these optimal policy responses with those which ignore parameter uncertainty allows us to draw some conclusions about the implications of parameter uncertainty for monetary policy.

Why focus only on uncertainty arising from sampling error in the parameter estimates? This question is best answered by referring to each of the remaining sources of forecast error. Firstly, structural change is the most difficult source of uncertainty to deal with because the extent to which the underlying structure of the economy is changing is virtually impossible to determine in the midst of those changes. Clements and Hendry have devoted considerable resources toward exploring the consequences of such underlying structural change. However, this type of analysis is only feasible in a controlled simulation. In the context of models that attempt to forecast actual economic data, for which the 'true' economic structure is not known, this type of exercise can only be performed by making assumptions about the types and magnitudes of structural shifts that are likely to impact upon the economy. With little or no information on which to base these assumptions, it is hazardous to attempt an empirical examination of their consequences for the conduct of monetary policy.[1]

Similarly, it is difficult to specify the full range of misspecifications that may occur in a model. Without being able to justify which misspecifications are possible and which are not, analysis of how these misspecifications affect the implementation of policy must be vague at best.

Turning to the third source of forecast errors, it is well known that when the policy-maker's objective function is quadratic, mean zero shocks affecting the system in a linear fashion have no impact on optimal policy until the shocks actually occur.[2] To the extent that a linear model is a sufficiently close approximation to the actual economy and given quadratic preferences, this implies that the knowledge that unforecastable shocks will hit the economy in the future has no effect on current policy. For this reason, the impact of random additive shocks to the economic system is ignored in this paper. Instead, we concentrate on multiplicative parameter uncertainty.

It is a common assumption in many policy evaluation exercises that the current state of the economy is accurately measured and known with certainty. However,

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1 However, it may be possible to exploit the work of Knight (1921) to generalise the expected utility framework for policy-makers who are not certain about the true structure and evolution of the economy. See, for example, Epstein and Wang (1994).

2 This is the certainty equivalence result discussed, for example, in Kwakernaak and Sivan (1972).

the initial state of the economy, which provides the starting point for forecasts, is often mismeasured, as reflected in subsequent data revisions. It is possible to assess the implications of this data mismeasurement for forecast uncertainty by estimating the parameters of the model explicitly taking data revisions into account (Harvey 1989). For example, in a small model of the US economy, Orphanides (1998) calibrated the degree of information noise in the data and examined the implications for monetary policy of explicitly taking this source of uncertainty into account. Because this type of evaluation requires a complete history of preliminary and revised data, it is beyond the scope of this paper.

In light of these issues, this paper focuses on parameter estimation error as the only source of uncertainty. From a theoretic perspective, this issue has been dealt with definitively by Brainard (1967). Brainard developed a model of policy implementation in which the policy-maker is uncertain about the impact of the policy instrument on the economy. With a single policy instrument (matching the problem faced by a monetary authority, for example) Brainard showed that optimal policy is a function of both the first and second central moments characterising the model parameters.[3] Under certain assumptions about the cross correlations between parameters, optimal policy under uncertainty was shown to be more conservative than optimal policy generated under the assumption that the true parameters are actually equal to their point estimates. However, Brainard also showed that it is possible for other assumptions about the joint distribution of the parameters to result in more active use of the policy instrument than would be observed if the policy-maker ignored parameter uncertainty.

Resolving the implications of parameter uncertainty for monetary policy then becomes an empirical issue. To this end, the next section describes the impact of this Brainard-type uncertainty on monetary-policy decision making in a small macroeconomic model of the Australian economy.

[3] Only the first and second moments are required because of the assumption that the policy-maker has quadratic preferences. Otherwise it is necessary to maintain the assumption that the parameters are jointly normally distributed.

3. An Economic Model

In this paper, rather than constructing a more sophisticated model, we use a simple model which has been developed in previous Reserve Bank research and which was recently applied by de Brouwer and Ellis (1998). Work on constructing a more realistic model for forecasting and policy analysis was beyond the scope of this paper.

The model we use is built around two identities and five estimated relationships determining key macroeconomic variables in the Australian economy: output, prices, unit labour costs, import prices and the real exchange rate. Despite its size, the model captures most aspects of the open economy monetary-policy transmission mechanism, including the direct and indirect influences of the exchange rate on inflation and output. The model is totally linear with only backward-looking expectations on the part of wage and price setters and financial market participants. The equations are estimated separately, using quarterly data from September 1980 to December 1997, except for the real exchange rate equation, which was estimated using quarterly data from March 1985.[4] The specification of the model is summarised in Table 1. The model is described in more detail in Appendix B of de Brouwer and Ellis (1998).

It is also necessary to specify the preferences of the policy-maker, in this paper, the monetary authority. Specifically, we assume that the policy-maker sets the profile of the policy instrument, the nominal cash rate (short-term interest rate), to minimise the intertemporal loss function:

$$Loss = E_t\left[\alpha\sum_{j=1}^{h} gap_{t+j}^2 + \beta\sum_{j=1}^{h}(\pi_{t+j}-\pi^*)^2 + \gamma\sum_{j=1}^{h}(i_{t+j}-i_{t+j-1})^2\right] \qquad (1)$$

where gap is the output gap (Table 1), π is the year-ended inflation rate of the underlying CPI; π^* is the inflation target specified in year-ended terms and E_t is the expectations operator conditional on information available at time *t*.

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4 Unless otherwise specified, the equations were estimated by ordinary least squares and, where necessary, the variance-covariance matrices of the coefficients were estimated using White's correction for residual heteroscedasticity and the Newey-West correction for residual autocorrelation.

Table 1: Specification of the Model

Output(a)

$$\Delta y_t = \alpha_1 - \underset{(0.04)}{0.23}(y_{t-1} + 0.10\, rer_{t-1} - 0.05\, nftot_{t-1}) + \underset{(0.04)}{0.27}\, y^{US}_{t-1} + \underset{(0.07)}{0.29}\, \Delta y^{US}_t + \underset{(0.04)}{0.01}\, \Delta y^{farm}_{t-1}$$

$$+ \underset{(0.04)}{0.01}\, \Delta y^{farm}_{t-2} - \underset{(0.09)}{0.09}\, r_{t-2} - \underset{(0.10)}{0.01}\, r_{t-3} + \underset{(0.08)}{0.05}\, r_{t-4} - \underset{(0.07)}{0.09}\, r_{t-5} - \underset{(0.01)}{0.07}\, r_{t-6}$$

Prices(b)

$$\Delta p_t = \alpha_2 + \underset{(0.02)}{0.04}(ulc_{t-1} - p_{t-1}) + \underset{(0.02)}{0.04}(p^{mp}_{t-1} - p_{t-1}) + \underset{(0.02)}{0.08}\, \Delta ulc_t + \underset{(0.01)}{0.02}\, \Delta p^{mp}_{t-3} + \underset{(0.01)}{0.12}\, gap_{t-3}$$

Unit labour costs(c)

$$\Delta ulc_t = \underset{(0.05)}{0.50}\, \Delta p_{t-1} + \underset{(0.05)}{0.50}\, \Delta p_{t-2} + \underset{(0.03)}{0.09}\, gap_{t-1}$$

Import prices(d)

$$\Delta p^{mp}_t = \alpha_4 - \underset{(0.01)}{0.18}(p^{mp}_{t-1} - p^{fmp}_{t-1} + ner_{t-1}) - \underset{(0.06)}{0.51}\, \Delta ner_t$$

Real exchange rate(e)

$$\Delta rer_t = \alpha_5 - \underset{(0.10)}{0.31}\, rer_{t-1} + \underset{(0.17)}{0.19}\, tot_{t-1} + \underset{(0.07)}{0.72}(r_{t-1} - r^*_{t-1}) + \underset{(0.13)}{1.39}\, \Delta tot_t$$

Nominal exchange rate

$$ner_t \equiv rer_t - p_t + p^*_t$$

Real interest rate

$$r_t \equiv i_t - (p_t - p_{t-4})$$

y	Real non-farm gross domestic product	p^{fmp}	Foreign price of imports (trade-weighted price of exports from Australia's trading partners expressed in foreign currency units)
rer	Real trade-weighted exchange rate	ner	Nominal trade-weighted exchange rate
$nftot$	Non-farm terms of trade	tot	Terms of trade
y^{us}	Real United States gross domestic product	r^*	Foreign real short-term interest rate (smoothed GDP-weighted average of real short-term interest rates in the US, Japan and Germany)
y^{farm}	Real farm gross domestic product	p^*	Foreign price level
r	Real (cash) interest rate	i	Nominal cash rate (policy instrument)
p	Treasury underlying consumer price index		
ulc	Underlying nominal unit labour costs		
p^{mp}	Tariff-adjusted import prices		
gap	Output gap (actual output less a production-function based measure of potential output)		

Notes: All variables except interest rates are expressed in log levels.

Figures in ( ) are standard errors (adjusted for residual heteroscedasticity or autocorrelation where necessary).

(a) The coefficients on the lagged level of *rer* and *nftot* were calibrated so that an equal simultaneous rise in both the terms of trade and the real exchange rate results in a net contraction of output in the long run.

(b) As specified, this equation implies linear homogeneity in the long-run relationship between prices, nominal unit labour costs and import prices (this restriction is accepted by the data).

(c) The restriction that the coefficients on lagged inflation sum to unity was imposed (this restriction is accepted by the data) and the equation was estimated by generalised least squares to correct for serial correlation.

(d) As specified, this equation implies linear homogeneity between the domestic price of imports, the nominal exchange rate and the foreign price of imports. This is equivalent to assuming purchasing power parity and full first-stage pass-through of movements in the exchange rate and foreign prices to domestic prices.

(e) A Hausman test failed to reject the null hypothesis that the first difference of the terms of trade was exogenous so results are reported using OLS estimates (OECD industrial production and lagged differences of the terms of trade were used as instruments when conducting the Hausman test).

The first two terms in this objective function describe the policy-maker's preference for minimising the expected output gap and deviations of expected inflation from target. The third term in the loss function represents the penalty attached to volatility in the policy instrument. This term is included to reduce the monetary authority's freedom to set policy in a way that deviates too far from the observed behaviour of the policy instrument. By penalising volatility in the policy instrument, this term imposes a degree of interest-rate smoothing (Lowe and Ellis 1997).

At each point in time, optimal policy is achieved by minimising the loss function with respect to the path of the policy instrument over the forecast horizon, $t+1$ to $t+h$, subject to the system of equations described in Table 1.

The coefficients, α, β and γ are the relative weights (importance) attached to minimisation of the output gap, deviations of inflation from target and movements in the policy instrument. In this paper, we set α, β and γ equal to 0.02, 0.98 and 0.02 respectively. This particular combination of weights characterises a monetary authority with a strong emphasis on keeping inflation close to target. The weights were selected so that the optimal policy response brings inflation back to target within a reasonable number of years for most common kinds of shocks. There is a much higher weight on inflation than on the output gap because the output gap is an important determinant of inflation. Therefore, policy which concentrates on getting inflation back to target also indirectly aims to close the output gap. While optimal policy is certainly sensitive to the choice of weights, they do not affect the qualitative implications of parameter uncertainty for monetary policy.

This completes the description of the model and the objectives of the policy-maker. The simulations in the following sections report optimal policy responses to shocks that move the model from its steady state, where the steady state is characterised by the error-correction terms in the equations.

The steady-state conditions are satisfied by normalising all of the constants and variables to zero and assuming that the exogenous variables have zero growth rates. This particular steady state has two advantages. First, the zero growth assumption eliminates the need for more sophisticated modelling of the exogenous variables.[5] Second, different parameter estimates imply different long-run relationships, but the zero-level steady state is the only one that satisfies each of these estimated long-run relationships. This means that the results which we present later can be interpreted as deviations from baseline.

4. Optimal Policy Ignoring Parameter Uncertainty

All of the estimated regression parameters in Table 1 are point estimates of the true parameter values and, as such, are random variables. The uncertainty surrounding these point estimates is partly reflected in their associated standard errors. This section highlights the consequences for monetary policy *when the policy-maker assumes that these point estimates accurately describe the true economy*. We generate a range of model-optimal policy responses and associated forecast profiles that would obtain under different draws of the parameter estimates from their underlying distribution. We describe these optimal policy responses as 'naive' because they ignore parameter uncertainty. In Section 6, we show how these policy responses can change when the optimal policy problem is solved recognising parameter uncertainty.

For each equation, we assume that the parameters are normally distributed with first moments given by their point estimates in Table 1 and second moments given by the appropriate entries in the estimated variance-covariance matrix of the parameter vector.[6] Because each equation is estimated separately, there is no information available concerning the cross correlations between the parameters in the different equations. This implies that the variance-covariance matrix of the

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[5] However, in practice, generating forecasts and optimal policy responses from this model would require explicit models for the exogenous variables, which would introduce additional parameter uncertainty.

[6] This approach to defining a distribution from which to draw the parameters of the model also ignores uncertainty about the estimates of the variance-covariance matrices themselves. Note also that the assumption that all parameter estimates are normally distributed is not correct. For example, the speed of adjustment parameters in each of the error-correction equations in Table 1 are actually distributed somewhere between a normal distribution and the Dickey-Fuller distribution (Kremers, Ericsson and Dolado 1992). This distinction is unlikely to make much difference to our results however, so for computational convenience, we have maintained the assumption that all of the parameters are normally distributed.

entire parameter vector is block diagonal, with each block given by the variance-covariance matrix of each individual equation.[7]

While we characterise parameter uncertainty as being entirely caused by sampling error, this understates the variety of factors that can contribute to imprecision in the parameter estimates. When we estimate each of the five behavioural equations, we assume that the parameters do not change over time. Any changes in the parameters must then be partially reflected in the parameter variance-covariance matrix. This contribution to parameter uncertainty actually derives from model misspecification rather than sampling error.[8] While we ignore these distinctions for the remainder of the paper, we acknowledge that the sampling error interpretation of the variance-covariance matrices may overstate the true sampling error problems and understate the problems of model misspecification and structural breaks in the model.

The remainder of this section presents forecasts that arise when the monetary authority faces a given parameter-estimate draw and believes that this draw represents reality, with no allowance made for uncertainty.[9] By solving the optimal-policy problem for a large number of parameter draws, we obtain a range of forecasts which indicate the consequences of ignoring parameter uncertainty. Starting from the steady state defined in the previous section, we assume that the system is disturbed by a single one percentage point shock to one of the five estimated equations. Then, for one thousand different parameter draws, the naive optimal policy problem is solved to generate the path of the nominal cash rate and the corresponding forecast profiles over a ten-year horizon. Each of the simulations that follow are based on the same one thousand draws from the underlying parameter distribution.

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7 For the real exchange rate and terms of trade parameters in the output equation, which have been calibrated, the appropriate terms in the variance-covariance matrix have been approximated by the corresponding terms in the variance-covariance matrix of the *unconstrained* (fully estimated) output equation.

8 An alternate interpretation is that instead of the parameters being fixed, but just estimated with error, the parameters may in fact vary stochastically (and, in this case, from a multivariate normal distribution). With this interpretation, however, the distinction between model misspecification and sampling error becomes less clear.

9 Each draw of the model parameters requires pre-multiplication of a vector of independent standard normal variates by the lower triangular Cholesky decomposition of the full variance-covariance matrix.

For example, consider a one percentage point shock to real output. Figure 1 summarises the results of this simulation. In this figure, the maximum, minimum and median along with the first and third quartiles illustrate the dispersion of forecast profiles generated by the different parameter draws. The central black line denotes the median, while the limits of the light shaded regions are the maximum and minimum. The darker shaded region corresponds to the inter-quartile range.

Note that the spread of forecasts around the median need not be symmetric. This is because asymmetries result from non-linearities in the way that the model parameters enter the construction of the forecasts. Although the model is linear in each of the variables, forecasts can be high-order polynomials in the lag coefficients.

To begin with, the output shock opens up a positive output gap which generates wage pressures in the economy. Feedback between wages and prices means that this wage pressure eventually feeds into price inflation. Consistent with the monetary authority's objectives, the optimal response to this shock is to initially raise the nominal cash rate. However, the size of this initial tightening can vary by up to three-quarters of a percentage point, depending on the parameter draw. With backward-looking inflation expectations, the rise in nominal interest rates raises the real cash rate, which has a dampening effect on output and eventually reverses the upward pressure on unit labour costs and inflation. The higher real interest rate also appreciates the real and nominal exchange rate, lowering inflation directly by reducing the Australian dollar price of imports and indirectly, by reducing output growth.

Over time, the initial tightening is reversed and eventually policy follows a dampening cycle as the output gap is gradually closed and wage and price inflation pressures subside. In the limit, all real variables and growth rates return to target and the system returns to the steady state.[10]

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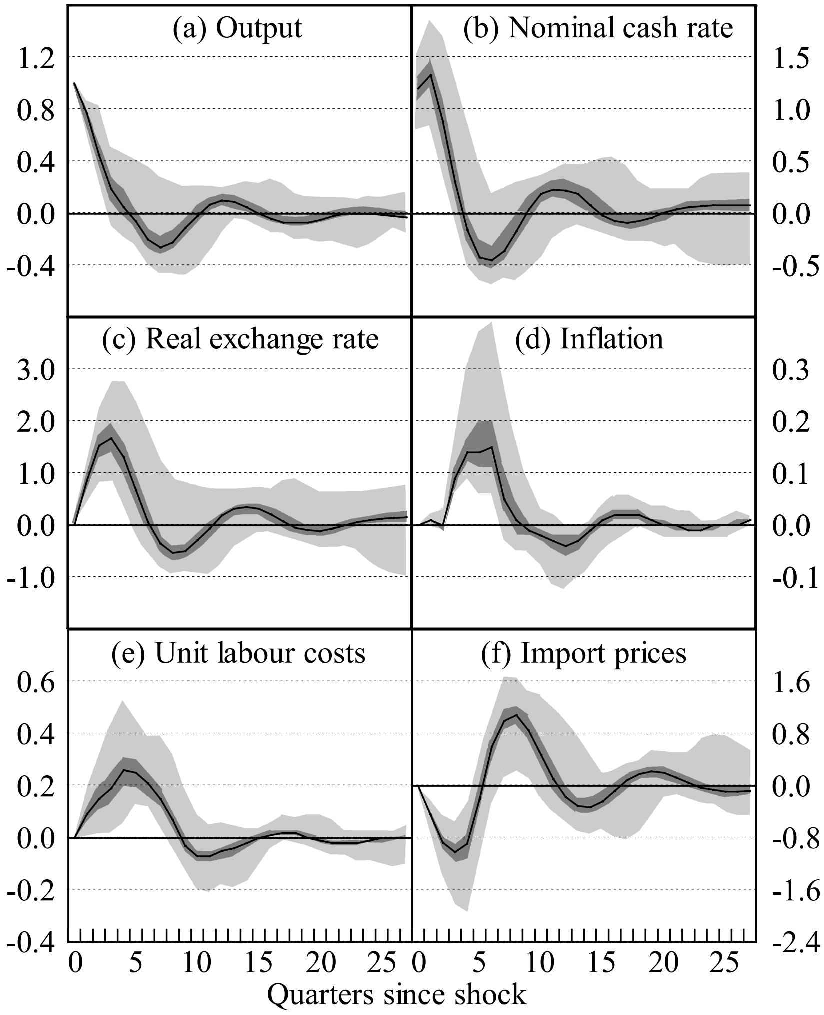
[10] While this is true for the model which we are using in this paper, after the 27 periods shown in Figure 1, some of the variables do not completely return to steady state. This is because the mean parameter draw results in a model which is quite persistent anyway and furthermore, some of the more extreme parameter draws can generate larger and more long-lasting cyclical behaviour in the variables. Eventually, however, all of the real variables and growth rates will return to steady state.

Figure 1: Real Output Shock

Deviations from equilibrium, percentage points



What is most striking about these simulations is the range of different forecast profiles caused by parameter uncertainty. For example, depending on the parameter draw, the optimal policy response at any time during the forecast horizon can vary by as much as one and a half percentage points. This variation demonstrates that naive policy responses are not robust across parameter draws. Faced with the observed parameter uncertainty in this model (captured by the variance-covariance matrix), there is no way of knowing *ex ante* which parameter draw is closest to the true parameters of the economy, and therefore, there is scope for a wide margin of error following implementation of any one of these optimal policy responses.

It should be stressed that the optimal policy responses in Figure 1 assume no learning on the part of the monetary authority. Although the monetary authority may set interest rates according to a calculated optimal policy path, the economy will only ever evolve according to the true parameter draw. Generically, the forecasts of the monetary authority will be proved wrong *ex-post*, providing a signal that the initial parameter estimates were incorrect. If the monetary authority learns more about the true model parameters from this signal, then Brainard-type uncertainty will gradually become less relevant over time.[11] However, in the naive policy responses shown in Figure 1, this type of learning is ruled out because we assume that the policy-maker always believes that the given parameter estimates are the true parameter values. In this case, any deviation between the actual and forecast behaviour of the economy would be attributed to unanticipated shocks.

We also examine the range of forecast profiles obtained under shocks to the other endogenous variables. Figure 2 shows the optimal response of the nominal cash rate to various other one percentage point shocks. These simulations are similar to that shown for the output shock in the sense that they all exhibit considerable variation in the optimal policy response across different parameter draws. However, in all cases, the optimal policy response drives the economy back into equilibrium with real variables trending back to their baseline values and nominal growth rates stabilising in accordance with the inflation target.

These simulations show that, where there is uncertainty regarding the true model parameters, the naive optimal policy response can vary quite considerably with observed parameter estimates. There are certainly considerable risks involved in implementing policy assuming that the estimated parameters are equal to their true values. In the next section, we demonstrate how the optimal policy problem can be modified to explicitly take into account parameter uncertainty. Rather than solving for the optimal path of the cash rate for a particular set of parameter estimates, the monetary authority takes into account the uncertainty associated with the distribution of parameter estimates and adjusts the policy response accordingly.

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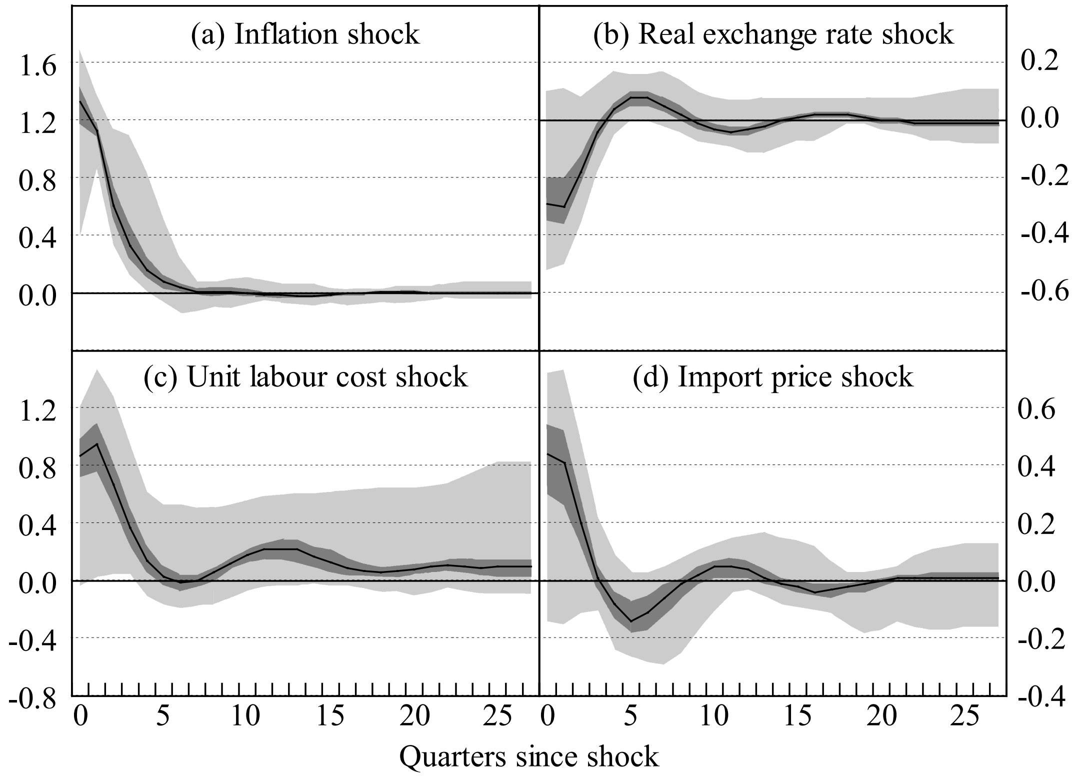
[11] Of course, the policy-maker will not be able to resolve uncertainty through time if the source of the *ex-post* forecasting error is parameter variation.

Figure 2: Optimal Interest Rate Responses Ignoring Parameter Uncertainty

Deviations from equilibrium, percentage points



5. Characterising Parameter Uncertainty

The main contribution of this paper is to solve for optimal policy in the context of an empirical model of the Australian economy, taking into account parameter uncertainty. To achieve this, we first generalise the Brainard formulation of optimal policy under uncertainty to accommodate multivariate models and multiple-period time horizons. We then draw out the intuition of this formulation using a stylised example. In the following section, we apply this formulation to the more fully-specified empirical model described in Section 3 in order to examine some of the implications of parameter uncertainty for monetary policy.

Matrix notation makes the following generalisation of the optimal policy problem and its solution considerably more transparent. The generalisation is derived in more detail in Appendix A.

The optimal policy problem for a monetary authority with quadratic preferences given by Equation (1) and a backward-looking multivariate model of the economy (that is linear in both the variables and shocks) can be written in the following general form:

$$\min_{\mathbf{R}} Loss = E[\mathbf{T}'\mathbf{\Omega}\mathbf{T}], \tag{2}$$

subject to:

$$\mathbf{T} = \mathbf{F}\mathbf{R} + \mathbf{G}, \tag{3}$$

where $\mathbf{T}$ is a vector of policy targets in each period of the forecast horizon; $\mathbf{R}$ is the vector of policy instruments; the matrices, $\mathbf{F}$ and $\mathbf{G}$ are functions of both history and the parameter estimates while $\mathbf{\Omega}$ summarises the penalties contained in the objective function. The time subscript has been omitted for simplicity. This general form for the optimal policy problem highlights its similarity to the problem originally solved by Brainard. Because the loss function is quadratic and the constraint set is linear, the usual optimal policy response under parameter uncertainty will apply.

Specifically, if $\mathbf{F}$ and $\mathbf{G}$ are stochastic, then the solution to the optimal policy problem is:

$$\tilde{\mathbf{R}}^* = -[E(\mathbf{F}'\mathbf{\Omega}\mathbf{F})]^{-1} E(\mathbf{F}'\mathbf{\Omega}\mathbf{G}), \tag{4}$$

which can also be expressed as:

$$\tilde{\mathbf{R}}^* = -\left(\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{F}} + E\left[\left(\mathbf{F}-\overline{\mathbf{F}}\right)'\mathbf{\Omega}\left(\mathbf{F}-\overline{\mathbf{F}}\right)\right]\right)^{-1}\left(\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{G}} + E\left[\left(\mathbf{F}-\overline{\mathbf{F}}\right)'\mathbf{\Omega}\left(\mathbf{G}-\overline{\mathbf{G}}\right)\right]\right), \tag{5}$$

where $\overline{\mathbf{F}}$ is the expectation of $\mathbf{F}$ and $\overline{\mathbf{G}}$ is the expectation of $\mathbf{G}$.

Alternatively, if $\mathbf{F}$ and $\mathbf{G}$ are deterministic (with values $\overline{\mathbf{F}}$ and $\overline{\mathbf{G}}$), then the solution to the problem becomes:

$$\mathbf{R}^* = -\left(\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{F}}\right)^{-1}\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{G}}. \tag{6}$$

$\mathbf{F}$ and $\mathbf{G}$ will be stochastic if they contain parameter estimates. Therefore, the solution described by Equations (4) and (5) corresponds to optimal policy acknowledging parameter uncertainty. The deterministic case in Equation (6) describes the naive policy response, when the monetary authority ignores parameter uncertainty. Comparing Equations (5) and (6), the difference between optimal policy responses with and without parameter uncertainty can be ascribed to the variance of $\mathbf{F}$ and the covariance between $\mathbf{F}$ and $\mathbf{G}$. Brainard's policy conservatism result depends crucially on the *independence* of $\mathbf{F}$ and $\mathbf{G}$. However, $\mathbf{F}$ and $\mathbf{G}$ will not be independent if they are derived from a model that exhibits persistence.

To make the optimal-policy definition in Equation (5) operational, it is necessary to compute the variance and covariance terms. This can be done using a sample estimate of the loss function in Equation (2):

$$Loss = \frac{1}{N}\sum_{i=1}^{N}\mathbf{T}_i'\mathbf{\Omega}\mathbf{T}_i\,, \tag{7}$$

where N is the number of parameter draws. This essentially computes the average loss over N parameter draws. The first-order necessary condition for this loss function to be minimised subject to the set of constraints in Equation (3) is then just the sample estimate of (4):

$$\hat{\mathbf{R}}^* = -\left(\frac{1}{N}\sum_{i=1}^{N}\mathbf{F}_i'\mathbf{\Omega}\mathbf{F}_i\right)^{-1}\left(\frac{1}{N}\sum_{i=1}^{N}\mathbf{F}_i'\mathbf{\Omega}\mathbf{G}_i\right). \tag{8}$$

By averaging across a large number of parameter draws, optimal policy can be computed from the linear relationship between the target variables of policy and the policy instrument. As the number of draws from the estimated parameter distribution increases, this approximation will tend to the true optimal policy given by Equation (4). The naive policy response is computed by setting N=1 and using the *mean* parameter draw to compute optimal policy from Equation (8). In this case, the mean parameter draw is interpreted as the 'true' model parameters, ignoring any uncertainty which we may have about them.

To illustrate the intuition behind these results, we first examine optimal policy responses in a stylised model which is an extension of the model considered by Brainard (1967). In the next section, we apply the generalised optimal policy solution to the model summarised in Table 1.

The original Brainard model assumed that the policy-maker minimised squared deviations of a variable y from a target (normalised to zero) by controlling a single policy instrument i, for an economy described by:

$$y_t = \theta i_t + \varepsilon_t, \tag{9}$$

where θ is an unknown parameter representing the effectiveness of policy and ε is an independently and identically distributed white-noise process. To explore the issues involved in generalising this specification, it is useful to consider an economy that also includes a role for dynamics:

$$y_t = \theta i_t + \rho y_{t-1} + \varepsilon_t, \tag{10}$$

where ρ is another unknown parameter representing the degree of persistence in the economy.

In this model, parameter uncertainty arises when ρ and θ can only be imprecisely estimated. The central message of this section and the next is that the implications of parameter uncertainty depend crucially upon the *relative* uncertainty about policy effectiveness (θ) and persistence (ρ). If uncertainty about policy effectiveness dominates, then the usual Brainard conservatism result obtains. However, if uncertainty about persistence is more important, then optimal policy may be more aggressive.

The loss function is assumed to be the unweighted sum of squared deviations of the target variable y in the current and all future periods. For a given shock to the target variable, the optimal policy response is found by minimising this loss function. Assume that the two parameter estimates in Equation (10) are drawn from different independent normal distributions. For the uncertainty-aware optimal policy, we take one thousand draws from the underlying parameter distributions and compute the optimal policy response using the frequency sampling approach described by Equation (8). We then compare this with the naive optimal policy response which is computed using only the mean parameter draw.

To contrast the policy implications of each of these two types of uncertainty, we derive the optimal policy responses to a standardised shock under differing levels of relative uncertainty about each of the parameters. In what follows, we assume that there is a one unit shock to the target variable in the initial period ($\varepsilon_0 = 1$), but thereafter ε is zero.

First, we examine the case where the persistence parameter is known with certainty, but the policy effectiveness parameter can only be estimated imprecisely.[12] Figure 3(a) shows that the uncertainty-aware optimal policy response to the shock in y is more conservative than the naive response. Consequently, using the mean parameter draw to forecast the target variable outcomes in Figure 3(b), the uncertainty-aware policy response means that the target variable takes longer to return to target. Of course, the actual *ex-post* outcome under both policy responses will depend upon the 'true' parameters, which need not coincide with the mean parameter draw.

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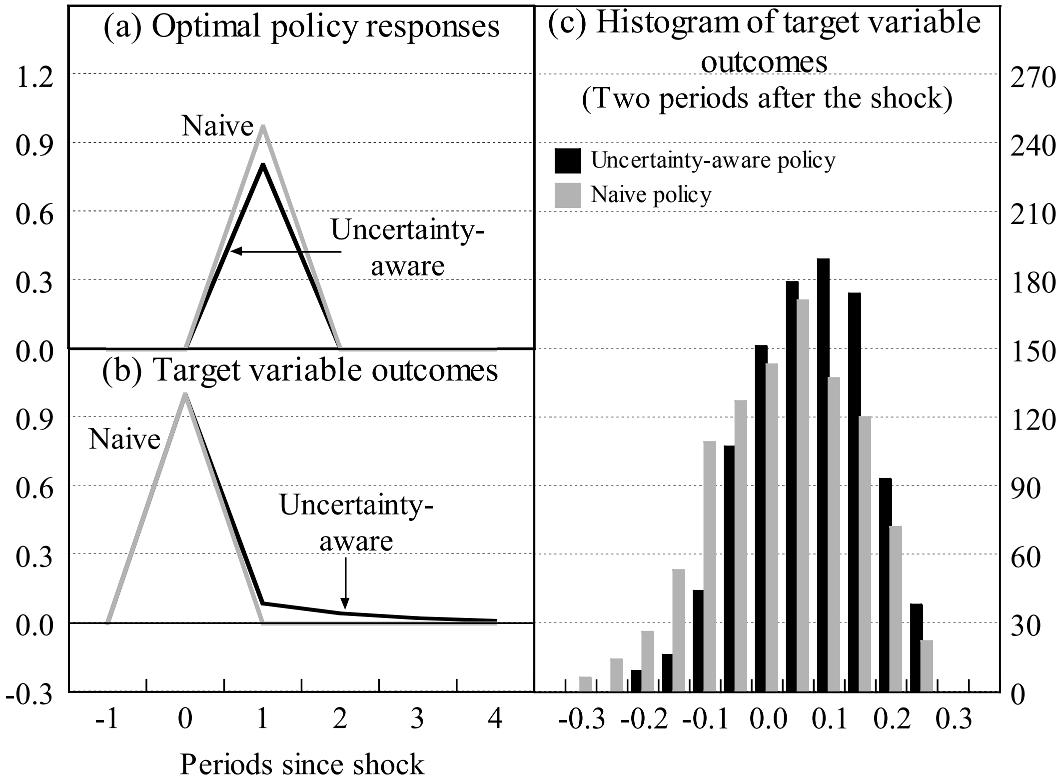
[12] Specifically, in this simulation we assume that the persistence parameter ρ takes the value 0.5 with certainty while the policy effectiveness parameter θ is drawn from a normal distribution with mean –0.5 and variance 0.25.

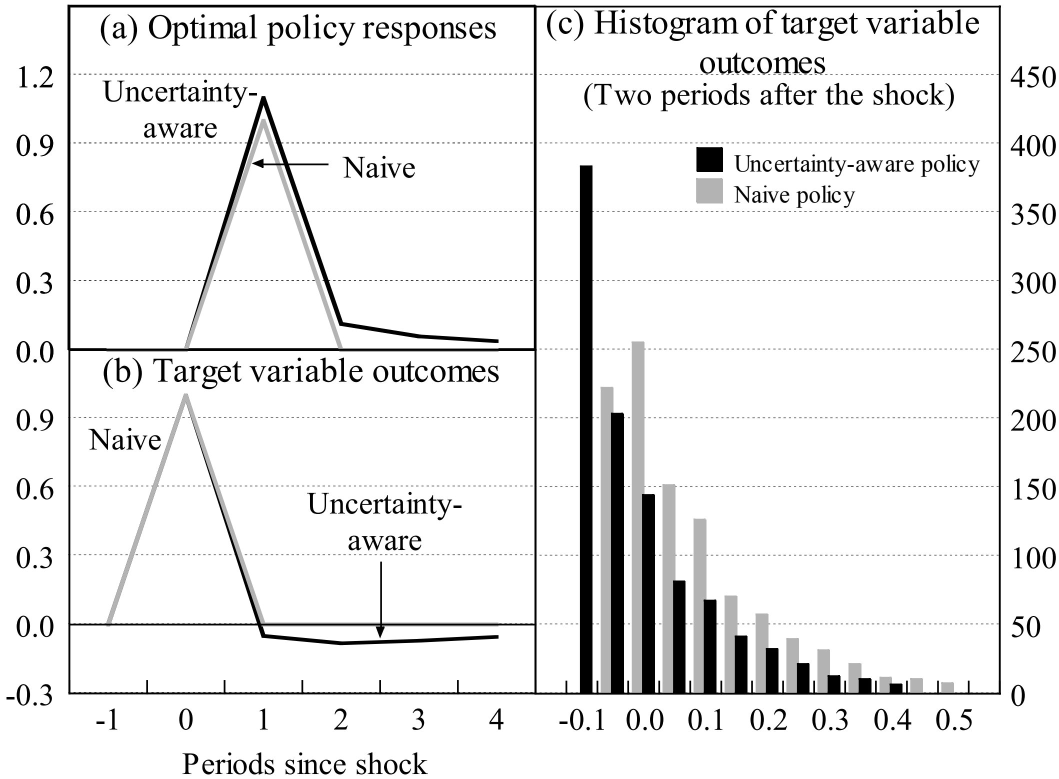
Figure 3: Uncertainty about Policy Effectiveness



For the two instrument paths shown in Figure 3(a), we can derive the implied *ex-post* target variable outcomes at any time horizon for each of the one thousand parameter draws. From this, we derive the distribution of target variable outcomes, which is presented as a histogram in Figure 3(c). In this Figure we only show the distribution of target variable outcomes *two* periods after the shock. As expected, under naive policy, the distribution of outcomes is approximately normally distributed with mean zero. For the conservative, uncertainty-aware optimal policy, however, the distribution of target variable outcomes for the one thousand parameter draws is slightly skewed and more tightly focused on small positive outcomes. In this example, because the conservative policy response reduces the spread of possible outcomes, it will generate a lower expected sum of squared deviations of y from target and dominates the naive policy response. This, of course, echoes Brainard's result; when there is uncertainty about the effects of policy then it pays to be more conservative with the policy instrument because the larger the change in the instrument, the greater will be the uncertainty about its final effect on the economy.

At the other extreme, we now consider the case where the effectiveness of policy is known with certainty, but the degree of persistence can only be imprecisely estimated.[13] Using the same one thousand parameter draws as before, the results in Figure 4(a) suggest that the uncertainty-aware optimal policy response is now more aggressive than the naive policy response. Figure 4(b) shows the target variable outcomes for these two policy responses using the mean parameter draw.

Figure 4: Uncertainty about Persistence



If the policy-maker ignores uncertainty, then the target variable will only follow the path shown in Figure 4(b) if the 'true' parameters coincide with the mean parameter draw. The target variable will overshoot, however, if persistence is lower than expected.[14] In this case, because the economy is less persistent than

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[13] In this case, θ takes the value –0.5 with certainty and ρ is drawn from a normal distribution with mean 0.5 and variance 0.25.

[14] Here an *overshooting* is defined as a negative target variable outcome because the initial shock was positive. An *undershooting*, then, is defined as a positive target variable outcome.

expected, the overshooting will be rapidly unwound, meaning that the shock is less likely to have a lasting impact on the economy. In contrast, if the target variable undershoots, then persistence must be higher than expected, so the effect of the shock will take longer to dissipate. A policy-maker that is aware of parameter uncertainty will take this asymmetry into account, moving the policy instrument more aggressively in response to a shock.[15] This will ensure that more persistent outcomes are closer to target at the cost of forcing less persistent outcomes further from target. This reduces the expected losses because the outcomes furthest from target unwind most quickly.

Generally speaking, when there is uncertainty about how persistent the economy is, that is, how a shock to y will feed into future values of y, then it makes sense to be more aggressive with the policy instrument with the hope of minimising deviations of the target variable as soon as possible. In Figure 4(c), for example, the aggressive uncertainty-aware policy response reduces the likelihood of undershooting, at the cost of increasing the chance of overshooting. This policy response dominates, however, because the loss sustained by the many small negative outcomes that it induces is more than offset by the greater loss sustained by the large positive outcomes associated with naive policy.[16]

These two simple examples show that the implications of parameter uncertainty are ambiguous. Policy should be more conservative when the effectiveness of policy is relatively imprecisely estimated, while policy may be more aggressive when the persistence of the economy is less precisely estimated. In between the two extreme examples which we have considered here, when there is uncertainty about all or most of the coefficients in a model, then one cannot conclude *a priori* what this entails for the appropriate interest-rate response to a particular shock. In the context of any estimated model, it remains an empirical issue to determine the implications of parameter uncertainty for monetary policy. This is what we do in the next section.

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[15] Of course, there is a limit on how aggressive policy can be before it causes worse outcomes.

[16] However, in this case, it is important to recognise that if the loss function contained a discount factor then this could reduce the costs of conservative policy. For example, with discounting, the naive policy-maker in Figure 4 will be less concerned about bigger actual losses sustained further into the forecast horizon. This result applies more generally; if policy-makers do not care as much about the future as the present, then they may prefer less activism rather than more.

This example also highlights the importance of assuming that monetary authorities never learn about the true model parameters. The monetary authority always observes the outcome in each period of time. If the extent of the policy error in the first period conveys the true value of ρ or θ, then policy could be adjusted to drive y to zero in the next period. Ruling out learning prevents these *ex-post* policy adjustments, making the initial policy stance time consistent. To the extent that uncertainty is not being resolved through time, this is a relevant description of policy. Additional data may sharpen parameter estimates but these gains are often offset by instability in the underlying parameters themselves. Sack (1997) explored the case in which uncertainty about the effect of monetary policy is gradually resolved through time by learning using a simple model in which the underlying parameters are initially chosen from a stochastic distribution.

6. Optimal Policy Acknowledging Parameter Uncertainty

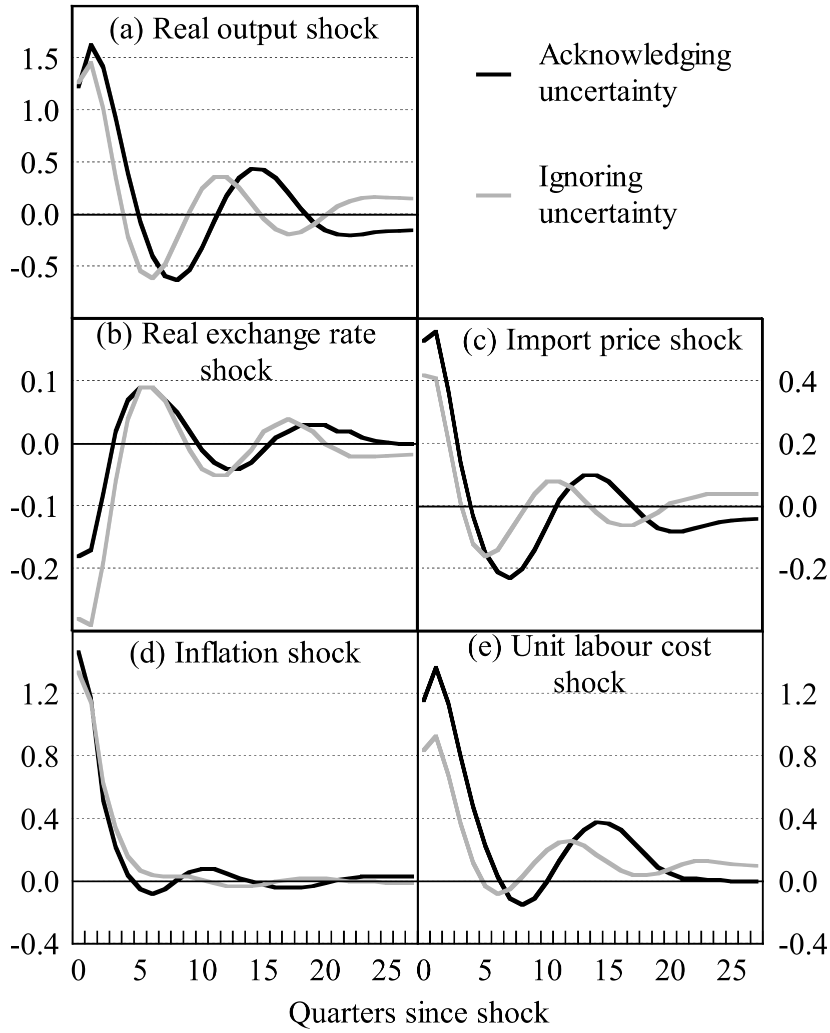
In this section we present uncertainty-aware optimal policy computations for the model described in Section 3 and compare them with naive optimal policy responses. The uncertainty-aware optimal policy response computed from Equation (8) is estimated using one thousand different draws from the underlying parameter distribution. A large number of draws is required because of the large number of parameters.

When computing the naive optimal policy response, it is necessary to specify the parameter vector that the monetary authority interprets as being the 'true' parameter values. Usually this vector would contain the original parameter estimates. However, we compute the naive policy response using the average value of the parameter vector draws. This prevents differences between the two policy profiles being driven by the finite number of parameter draws, the average of which may not coincide exactly with the original parameter draw.

Beginning with a one percentage point shock to output, Figure 5(a) compares optimal policy responses using the model described in Section 3. The darker line represents optimal policy when parameter uncertainty is taken into account while the lighter line represents the naive optimal policy response.

Figure 5: Optimal Interest Rate Responses Ignoring and Acknowledging Parameter Uncertainty

Deviations from equilibrium, percentage points



The key feature of interest in Figure 5(a) is the larger initial policy response when uncertainty is taken into account. While later oscillations in the cash rate are similar in magnitude, the early response of the cash rate is somewhat larger when policy takes parameter uncertainty into account. The finding that policy should react more aggressively because of parameter uncertainty is specific to the output shock used to generate Figure 5(a). For example, the naive policy response is relatively more aggressive for a one percentage point shock to the real exchange rate, as shown in Figure 5(b). This suggests that the persistence of a real exchange rate shock is more precisely estimated than the persistence of an output shock relative to the estimated effectiveness of policy for each of these shocks. With less relative uncertainty about the persistence of a real exchange rate shock, the optimal policy response taking into account uncertainty is more conservative because it is dominated by uncertainty about policy effectiveness.

Figures 5(c) - 5(e) present the policy responses to shocks to import prices, consumer prices and unit labour costs respectively. In each case, the naive policy response is initially more conservative than the policy response which takes account of parameter uncertainty.

These three types of nominal shocks to the model confirm that conservatism is by no means the generic implication of parameter uncertainty. In our model, it appears that uncertainty about the effectiveness of the policy instrument is generally dominated by uncertainty about model persistence and this explains the more aggressive optimal policy response to most of the shocks which we examined.

7. Conclusion

This paper extends Brainard's formulation of policy-making under uncertainty in several directions. First, it generalises the solution of the optimal policy problem to accommodate multiple time-periods and multiple objectives for policy. This generalisation develops the stochastic properties of the equations relating target variables to the policy instrument from the estimated relationships defining the underlying economic model.

Whereas uncertainty about the effectiveness of monetary policy tends to recommend more conservative policy, we explore the intuition for why other forms of parameter uncertainty may actually lead to more aggressive policy. In a simple example, we show that uncertainty about the dynamics of an economy can be a source of additional policy activism. However, this consequence of parameter uncertainty is only relevant in a multi-period generalisation of the Brainard model.

In the context of any specific model, it is an empirical issue to determine the exact implications of parameter uncertainty for monetary policy. We examine this using a small linear model of the Australian economy that captures the key channels of the monetary policy transmission mechanism within an open-economy framework. Optimal policy responses ignoring parameter uncertainty are compared with optimal responses that explicitly take parameter uncertainty into account. While the differences between these policy responses vary with the source of shocks to the economy, our evidence suggests that, for most shocks in our model, parameter uncertainty motivates somewhat more aggressive use of the instrument.

Although the results in this paper are reported as deviations from equilibrium, the method used to construct optimal policy responses under parameter uncertainty is also applicable in a forecasting environment where past data must be taken into account. The approach is applicable to all backward-looking linear models in which the objectives of the policy-maker are quadratic.

The simulations also demonstrate how frequency-sampling techniques can be used to evaluate the analytic expression for optimal policy under parameter uncertainty, despite the presence of complex expectations terms. This approach to policy determination is as practical, and more theoretically appealing, than the application of alternative rules-based approaches.

While the findings of the paper are of considerable interest, they should not be overstated. In particular, the implications of parameter uncertainty are dependent upon the type of shock being accommodated. They are also dependent upon the specification of the model. For example, changes to the model specification could substantially alter the measured uncertainty attached to the effectiveness of policy relative to the measured uncertainty associated with the model's dynamics. If the techniques developed in this paper are to be of wider use, the underlying model must first be well understood and carefully specified. Also, it is worth remembering that, although our model suggests that optimal monetary policy taking account of uncertainty is more activist for most kinds of shocks, the difference in policy response is quite small relative to the degree of conservatism that is actually practiced by most central banks.

In this paper we have not sought to argue that conservative monetary policy is not optimal. In fact, there are probably a number of good reasons why conservative policy may be optimal. Instead, the central message of the paper is that, if we are to motivate conservative monetary policy, then explanations other than Brainard's are required.

Appendix A: Generalising the Optimal Policy Problem

In this appendix, we generalise Brainard's (1967) solution to the optimal policy problem for a monetary authority with quadratic preferences using a dynamic, multivariate model with stochastic parameter uncertainty.

To begin with, we show how the optimal policy problem for a monetary authority with quadratic preferences given by Equation (1) and a backward-looking multivariate model of the economy (that is linear in both the variables and the shocks) can be written in the following general form:

$$\min_{\mathbf{R}} Loss = E\left[\mathbf{T}'\mathbf{\Omega}\mathbf{T}\right], \tag{A1}$$

subject to:

$$\mathbf{T} = \mathbf{F}\mathbf{R} + \mathbf{G}. \tag{A2}$$

To prove this, recall that the preferences of the monetary authority can be summarised by the following intertemporal quadratic loss function:

$$Loss = E_t\left[\alpha\sum_{j=1}^{h} gap_{t+j}^2 + \beta\sum_{j=1}^{h}(\pi_{t+j} - \pi^*)^2 + \gamma\sum_{j=1}^{h}(i_{t+j} - i_{t+j-1})^2\right], \tag{A3}$$

which can be rewritten using matrix notation as:

$$Loss = E_t\left[\alpha\,\mathbf{Y}_t'\mathbf{Y}_t + \beta\,\mathbf{\Pi}_t'\,\mathbf{\Pi}_t - \gamma\,\mathbf{R}_t'(\mathbf{I}-\mathbf{\Gamma})'(\mathbf{I}-\mathbf{\Gamma})\mathbf{R}_t\right] \tag{A4}$$

where:

$$\mathbf{Y}_t = \begin{bmatrix} gap_{t+1} & gap_{t+2} & \dots & gap_{t+h} \end{bmatrix}', \tag{A5}$$

$$\mathbf{\Pi}_t = \left[\left(\pi_{t+1} - \pi^*\right) \;\; \left(\pi_{t+2} - \pi^*\right) \;\; \dots \;\; \left(\pi_{t+h} - \pi^*\right)\right]', \tag{A6}$$

$$\mathbf{R}_t = \begin{bmatrix} i_{t+1} & i_{t+2} & \dots & i_{t+h} \end{bmatrix}', \tag{A7}$$

$\mathbf{\Gamma}$ is the matrix that lags the nominal interest rate vector $\mathbf{R}_t$ by one period; and $\mathbf{I}$ is an $(h \times h)$ identity matrix. The subscript t denotes the current date from which forecasts are being generated.

Given that the model of the economy (Table 1) is linear, the policy target variables are affine transformations of the forecast profile for the policy instrument:

$$\mathbf{Y}_t = \mathbf{A}\mathbf{R}_t + \mathbf{B} \tag{A8}$$

and

$$\mathbf{\Pi}_t = \mathbf{C}\mathbf{R}_t + \mathbf{D}, \tag{A9}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are stochastic matrices constructed from the parameters of the model and the history of the economy. The structure of these stochastic matrices is determined by the relationships laid out in the definition of the model's equations. Matrices $\mathbf{B}$ and $\mathbf{D}$ are the impulse response functions of the output gap and inflation to a shock at time *t*. Likewise, matrices $\mathbf{A}$ and $\mathbf{C}$ are the marginal impact of the nominal cash rate on the output gap and inflation respectively.

By defining $\mathbf{\Delta}_t \equiv (\mathbf{I} - \mathbf{\Gamma})\mathbf{R}_t$ as the vector of first differences in the nominal cash rate over the forecast horizon, it is possible to specify the full set of policy targets as:

$$\mathbf{T}_t = \left[\mathbf{Y}_t' \quad \mathbf{\Pi}_t' \quad \mathbf{\Delta}_t'\right]. \tag{A10}$$

Then, upon dropping time subscripts, the optimal policy problem can be restated succinctly as:

$$\min_{\mathbf{R}} Loss = E\left[\mathbf{T}'\mathbf{\Omega}\mathbf{T}\right], \tag{A11}$$

subject to:

$$\mathbf{T} = \mathbf{F}\mathbf{R} + \mathbf{G}, \tag{A12}$$

where the matrices $\mathbf{F}$ and $\mathbf{G}$ are defined in terms of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $(\mathbf{I} - \mathbf{\Gamma})$ as:

$$\mathbf{F} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & \mathbf{C} & 0 \\ 0 & 0 & \mathbf{I}-\mathbf{\Gamma} \end{bmatrix}$$
$$\mathbf{G} = \begin{bmatrix} \mathbf{B} \\ \mathbf{D} \\ 0 \end{bmatrix} \tag{A13}$$

and the weights on the different components of the loss function α, β and γ, have been subsumed into the diagonal matrix $\mathbf{\Omega}$ according to:

$$\mathbf{\Omega} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \otimes \mathbf{I} \tag{A14}$$

where $\mathbf{I}$ is the same identity matrix used to define the first differences in the cash rates, $\mathbf{\Delta}$. Ignoring the fact that it is in matrix notation and observing that the target values of the target variables have been normalised to zero, this problem is exactly the same as that examined by Brainard (1967).

If $\mathbf{F}$ and $\mathbf{G}$ are stochastic, the solution to the optimal policy problem described by Equations (A1) and (A2) is:

$$\tilde{\mathbf{R}}^* = -[E(\mathbf{F}'\mathbf{\Omega}\mathbf{F})]^{-1} E(\mathbf{F}'\mathbf{\Omega}\mathbf{G}), \tag{A15}$$

which can also be expressed as:

$$\tilde{\mathbf{R}}^* = -\left(\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{F}} + E\left[\left(\mathbf{F}-\overline{\mathbf{F}}\right)' \mathbf{\Omega} \left(\mathbf{F}-\overline{\mathbf{F}}\right) \right] \right)^{-1} \left(\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{G}} + E\left[\left(\mathbf{F}-\overline{\mathbf{F}}\right)' \mathbf{\Omega} \left(\mathbf{G}-\overline{\mathbf{G}}\right) \right] \right). \tag{A16}$$

Alternatively, if $\mathbf{F}$ and $\mathbf{G}$ are deterministic (with values $\overline{\mathbf{F}}$ and $\overline{\mathbf{G}}$), then the solution to the optimal policy problem is:

$$\mathbf{R}^* = -\left(\overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{F}}\right)^{-1} \overline{\mathbf{F}}'\mathbf{\Omega}\overline{\mathbf{G}}. \tag{A17}$$

To show this, rewrite the loss function in Equation (A1) by adding and subtracting the expected values of $\mathbf{T}$ from it, yielding:

$$\min_{\mathbf{R}} Loss = E\left[(\mathbf{T}-\overline{\mathbf{T}}+\overline{\mathbf{T}})'\boldsymbol{\Omega}(\mathbf{T}-\overline{\mathbf{T}}+\overline{\mathbf{T}})'\right]. \tag{A18}$$

Upon expanding, this loss function can be also be expressed as:

$$\begin{aligned}\min_{\mathbf{R}} Loss &= E\left[(\mathbf{T}-\overline{\mathbf{T}})'\boldsymbol{\Omega}(\mathbf{T}-\overline{\mathbf{T}})\right]+\overline{\mathbf{T}}'\boldsymbol{\Omega}\overline{\mathbf{T}}+2\overline{\mathbf{T}}'\boldsymbol{\Omega}\, E\left[\mathbf{T}-\overline{\mathbf{T}}\right]\\ &= E\left[(\mathbf{T}-\overline{\mathbf{T}})'\boldsymbol{\Omega}(\mathbf{T}-\overline{\mathbf{T}})\right]+\overline{\mathbf{T}}'\boldsymbol{\Omega}\overline{\mathbf{T}},\end{aligned} \tag{A19}$$

taking advantage of the fact that $E(\mathbf{T}) \equiv \overline{\mathbf{T}}$.

Substituting in Equation (A2) and simplifying then yields:

$$\begin{aligned}\min_{\mathbf{R}} Loss = E\left[((\mathbf{F}-\overline{\mathbf{F}})\mathbf{R}+\mathbf{G}-\overline{\mathbf{G}})'\boldsymbol{\Omega}((\mathbf{F}-\overline{\mathbf{F}})\mathbf{R}+\mathbf{G}-\overline{\mathbf{G}}\right]+\mathbf{R}'\overline{\mathbf{F}}'\boldsymbol{\Omega}\overline{\mathbf{F}}\mathbf{R}\\ +\mathbf{2R}'\overline{\mathbf{F}}'\boldsymbol{\Omega}\overline{\mathbf{G}}+\overline{\mathbf{G}}'\boldsymbol{\Omega}\overline{\mathbf{G}}.\end{aligned} \tag{A20}$$

The first order necessary condition for this optimisation problem is obtained by differentiating with respect to $\mathbf{R}$:

$$2(\mathbf{F}-\overline{\mathbf{F}})'\boldsymbol{\Omega}(\mathbf{F}-\overline{\mathbf{F}})\widetilde{\mathbf{R}}^*+2(\mathbf{F}-\overline{\mathbf{F}})'\boldsymbol{\Omega}(\mathbf{G}-\overline{\mathbf{G}})\widetilde{\mathbf{R}}^*+2\overline{\mathbf{F}}'\boldsymbol{\Omega}\overline{\mathbf{F}}\widetilde{\mathbf{R}}^*+2\overline{\mathbf{F}}'\boldsymbol{\Omega}\overline{\mathbf{G}}=0. \tag{A21}$$

Solving for $\widetilde{\mathbf{R}}^*$ then gives optimal policy when taking uncertainty into account, as expressed in Equations (A15) and (A16). Given that the loss function is strictly convex, this first order necessary condition is also sufficient for a minimum of the expected loss function.

The naive optimal policy response shown in Equation (A17) obtains as a simplification of Equation (A16) when $\mathbf{F}$ is set to $\overline{\mathbf{F}}$ and $\mathbf{G}$ is set to $\overline{\mathbf{G}}$, that is, when $\mathbf{F}$ and $\mathbf{G}$ are deterministic.

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