

The Nonlinear Phillips Curve and Inflation Forecast Targeting

-Symmetric versus Asymmetric Monetary Policy Rules

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Abstract

We extend the Svensson (1997a) inflation forecast targeting framework with a convex Phillips curve. We derive an asymmetric target rule, that implies a higher level of nominal interest rates than the Svensson (1997a) forward looking version of the reaction function popularised by Taylor (1993). Extending the analysis with uncertainty about the output gap, we find that uncertainty induces a *further* upward bias in nominal interest rates. Thus, the implications of uncertainty for optimal policy are the opposite of standard multiplier uncertainty analysis.

Keywords: inflation targets, nonlinearities, asymmetries, stochastic control

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1 Introduction⁽²⁾

The 1990s saw the introduction of explicit inflation targets for monetary policy in a number of countries viz. New Zealand, Canada, the United Kingdom, Sweden, Finland and Spain. Inflation targeting has been introduced as a way of further reducing inflation and to influence market expectations, after disappointment with monetary targeting (New Zealand and Canada) or fixed exchange rates (United Kingdom, Sweden and Finland).

The relation between inflation targets and central bank preferences has been thoroughly investigated. On the one hand there is a theoretical literature Walsh (1995), Svensson (1997) that concludes that inflation targets can be used as a way of overcoming credibility problems because they can mimic optimal performance incentive contracts.³

On the other hand there is an empirical literature that looks whether inflation targets have been instrumental in reducing the policy-implied short-term trend rate of inflation [Leiderman and Svensson (1995)]. Broadly speaking the evidence is that inflation targets have indeed brought about a change in policy maker's inflation preferences.

Unlike the relation between inflation targets and central bank preferences, a relatively underexplored issue is how to translate inflation targets into short-term interest rates. This is the issue of how to map explicit *targets* for monetary policy into monetary policy *instruments*, or how to *implement* an inflation targeting framework. An exception is a recent, and important contribution by Svensson (1997a). Svensson shows that - because of lags in the transmission process of short-term interest rates to inflation - inflation targeting implies inflation *forecast* targeting. In his analysis the central bank's forecast becomes an explicit intermediate target and its optimal reaction function has the same form as the Taylor rule (1993)⁴. Recently, Clarida, Gali and Gertler (1997b) have shown that this type of reaction function does quite a good job of characterising monetary policy for the G3. The kind of rule that emerges is what they call "soft-hearted" inflation targeting. In response to a rise in expected inflation relative to target, each central bank raises nominal interest rates sufficiently enough to push up real rates, but there is a modest pure stabilisation component to each rule as well.

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³ This literature is surveyed in Schaling (1995). Also by increasing the accountability of monetary policy inflation targeting may reduce the inflation bias of discretionary policy. See Svensson (1997) and Nolan and Schaling (1996).

⁴ For an interesting recent study of the Taylor rule in a UK context see Stuart (1996).

Also, the 1990s have seen the development of the literature on the so-called nonlinear Phillips curve. [Chada, Masson and Meredith (1992), Laxton, Meredith and Rose (1995), Clark, Laxton and Rose (1995,1996), and Bean (1996)]. This recent literature puts the time-honoured inflation output trade-off debate in a fresh perspective by allowing for convexities in the transmission mechanism between the output gap and inflation. More specifically, according to this literature positive deviations of aggregate demand from potential (the case of an upswing or ‘boom’) are *more inflationary* than negative deviations (downswings) are *disinflationary*.⁵

This paper marries both strands of literature. We extend the Svensson (1997a) inflation forecast targeting framework with a convex Phillips curve. Using optimal control techniques we derive an asymmetric policy rule, that implies higher nominal interest rates than the Svensson (1997a) forward looking version of the reaction function popularised by Taylor (1993). This means that - if the economy is characterised by asymmetries - the Svensson (1997a) linear target rule may underestimate the correct level of interest rates.

The paper is organised into four remaining sections followed by one appendix. In Section 2 we present the model. In Section 3 we present the asymmetric policy rule in the deterministic case. In Section 4 we extend the analysis with uncertainty about the output gap. Section 5 confronts the implications of multiplicative parameter uncertainty for policy with those of the classic Brainard (1967) analysis. Section 6 concludes, the appendix provides proofs behind key results.

2 A Nonlinear Phillips Curve

As stated by Laxton et al (1995, pp 345-346) the broad acceptance of the expectations-augmented Phillips curve - and the associated “natural rate” hypothesis - led to the important conclusion that a long-run trade-off between activity and inflation did not exist. Subsequent research on output-inflation linkages has focused on how expectations are formed and the reasons for price “stickiness” that causes real variables to respond to nominal shocks. Almost all of this work, however, has been predicated on the assumption that the trade-off between activity and inflation is *linear*, that is, the response of inflation to a positive gap between actual and potential output is identical to a negative gap of the same size. Though analytically convenient, the linear model ignores much of the historical context underlying the original split between classical and Keynesian economics: under conditions of full employment, inflation appeared to respond strongly to demand conditions, while in deep recessions, it was relatively insensitive to changes in activity.⁶

⁵ In addition there is the view that the Phillips curve is *concave* Stiglitz (1997). It can be modeled by changing the sign of \mathbf{j} in equation (2.1). Obviously, all policy conclusion are reversed.

⁶ Indeed, as pointed out by Laxton et. al (1995), the original article by Phillips emphasised such an asymmetry, with excess demand having had a much stronger effect in raising inflation than excess supply had in lowering it.

Many of the tests that have been performed to test for nonlinearity have been uninformative because the filters that people have chosen have simply been fundamentally inconsistent with the existence of convexity. However, when properly tested, there is some evidence for asymmetries. Laxton et al. (1995) find that by pooling data from the major seven OECD countries the Phillips curve is nonlinear. Clark et al. (1996) find that the US inflation-output trade-off is also nonlinear, using quarterly data from 1964-1990. Debelle and Laxton (1997) find that the unemployment-inflation trade-off is nonlinear in the United Kingdom, the US and Canada. Finally, recent research at the Bank of England [Fisher et al. (1997)] also finds that a Phillips curve that embodies a mild asymmetry is consistent with UK data.

2.1 Nonlinear Output Inflation Dynamics

The main purpose of this Section is to combine a convex Phillips curve along the lines of Laxton, Meredith, and Rose (1995) with the Svensson (1997a) model of inflation targeting to allow for lags in the transmission process of short-term interest rates. Next, we use this model to analyse the effects of delaying monetary policy measures on the future levels of inflation and nominal interest rates. The functional form we employ to represent the nonlinearity in the inflation-output relationship is

$$\Delta p_{t+1} = f(\bullet) = \frac{\mathbf{a}_1 y_t}{1 - \mathbf{a}_j y_t} \quad (2.1)$$

where \mathbf{p} is $p_t - p_{t-1}$ is the inflation (rate) in year t , p_t is the (log) price level, y is an endogenous variable output, and $\mathbf{a}_1 > 0$ and $0 \leq \mathbf{j} < 1$ are parameters, and Δ is the backward difference operator. We normalise the natural rate of output *in the absence of uncertainty* to zero⁷. This means that y is the (log) of output relative to potential i.e. the output gap. Equation (2.1) is graphed in Figure 2.1. Its relevant properties can be derived by looking at the first derivative of $f(\bullet)$ - that is, the slope of the output inflation trade-off:

⁷ With uncertainty the natural rate of output in the nonlinear model, will always be *below* that of the linear model. See for instance Clark et al (1995). The reason is that if output were maintained, on average, equal to the natural rate of the linear model, then the asymmetry in the response of inflation to demand shocks would make it impossible to maintain inflation at a constant inflation target. To see this formally lead the Phillips curve one period and take expectations at time t , this yields $E_t \Delta p_{t+2} = E_t [\mathbf{a}_1 y_{t+1} / 1 - \mathbf{a}_j y_{t+1}]$. In a sustainable equilibrium with a constant rate of inflation equal to the inflation target, $E_t \Delta p_{t+2} = 0$. Taking account of Jensen's inequality we get

$0 = f(E_t y_{t+1}) + \mathbf{j} / 2 f''(E_t y_{t+1}) \mathbf{s}_e^2$. This equality then (implicitly) defines $E_t y_{t+1}$, the average level of output in the presence of shocks. With the convexity parameter value used in this paper ($\mathbf{j} = 0.5$) this level lies about 0.1 percent below the corresponding level of output in the absence of shocks. Since several empirical papers - see for instance Debelle and Laxton (1997) - suggest a larger gap between the stochastic and deterministic equilibrium.

$$f'(\bullet) = \frac{\mathbf{a}_1}{[1 - \mathbf{a}_j y_t]^2} \quad (2.2)$$

Following Laxton, Meredith and Rose (1995, pp. 349-350), it is useful to consider the limiting values of $f(\bullet)$ and its derivative for some specific values of \mathbf{j} and y

$$\lim_{j \rightarrow 0} f'(\bullet) = \mathbf{a}_1 \quad (2.3.a)$$

$$\lim_{y \rightarrow \frac{1}{\mathbf{a}_j}} f'(\bullet) = \infty, f(\bullet) = \infty \quad (2.3.b)$$

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$$\lim_{y \rightarrow -\infty} f'(\bullet) = 0, f(\bullet) = \frac{-1}{\mathbf{j}} \quad (2.3.c)$$

$$f'(0) = \mathbf{a}_1, f(0) = 0 \quad (2.3.d)$$

Equation (2.3.a) shows that, as the parameter \mathbf{j} becomes very small, the Phillips curve approaches a linear relationship, hence as in Bean (1996), the parameter \mathbf{j} indexes the curvature.

Equation (2.3.b) indicates that the effect on next year's inflation rises without bound as output approaches $1 / \mathbf{a}_j$. Hence, as in Chada, Masson and Meredith (1992) - henceforth CMM - $1 / \mathbf{a}_j$ represents an *upper bound* (henceforth y_{\max}) beyond which output cannot increase in the short run. Having described the Phillips curve it remains to specify the evolution of output. Following Svensson (1997a, p. 1115), we

assume that output is serially correlated, decreasing in the short -term interest rate and increasing in an exogenous demand shock x .

$$y_{t+1} = \mathbf{b}_1 y_t - (i_t - \mathbf{p}_t) + x_{t+1} \quad (2.4)$$

Where $0 < \mathbf{b}_1 < 1$ As can be seen from equations (2.1) and (2.4), the real base rate affects output with a one-year lag, and hence inflation with a two year lag, the control lag in the model.⁸

The exogenous variable is also serially correlated and assumed to be subject to a random disturbance \mathbf{e}_{t+1} not known at time t .

$$x_{t+1} = \mathbf{b}_2 x_t + \mathbf{e}_{t+1} \quad (2.5)$$

2.2 Optimal Monetary Policy

As in Svensson (1997a) monetary policy is conducted by a central bank with an inflation target \mathbf{p}^* (say 2.5 percent per year). We interpret inflation targeting as implying that the central bank's objective in period t is to choose a sequence of current and future interest rates $\{i_t\}_{t=t}$ such that

$$\underset{\{i_t\}}{\text{Min}} E_t \sum_{t=t}^{\infty} \mathbf{d}^{t-t} \left[\frac{(\mathbf{p}_t - \mathbf{p}^*)^2}{2} \right] \quad (2.6)$$

where the discount factor \mathbf{d} fulfills $0 < \mathbf{d} < 1$ and the expectation is conditional on the central bank's information set, Ω_t , that contains current (predetermined) output and inflation, its forecast of the demand shock and its perception of the asymmetry in the economy \mathbf{j} .¹⁰ Thus the central bank wishes to minimise

⁸ In case of *rational* expectations w.r.t. inflation in equation (2.4), what happens is the following. Through the Phillips curve (2.1) it can be seen that inflation at time $t + 1$ depends on the $f(\bullet)$ function. This means that - with model consistent expectations - expected inflation responds in a nonlinear fashion to the output gap as well. More specific, a positive output gap will increase *expected* inflation by more than a negative gap will reduce it. Of course, this implies that *ex ante* real rates now also respond asymmetrically. This add-on effect will thus reinforce the transmission effects of the asymmetry of the Phillips curve.

⁹ It is not really necessary to specify a distribution as long as it is assumed that this has *finite* support. This is necessary because by inverting the Philips curve it can be seen that output will hit the constraint if inflation goes to infinity. Now, with inflation targeting (that serves as a natural brake on the expansion of output) and (appropriately specified) finite support of shocks inflation will always be close enough to the target to prevent output hitting the capacity constraint.

¹⁰ Note that here the central bank is conducting monetary policy from a clear *forward looking* perspective. This means that - as elegantly stated by Greenspan in his Congressional testimony on 22 February 1995 - 'monetary policy will have a better chance of contributing to meeting the nation's macroeconomic objectives if we look forward as we act, however *indistinct* our view of the road ahead. Thus, over the past year [1994] we have firmed policy to head off inflation pressures not yet evident in the data.' Now, an interesting parallel can be drawn. *If* policy takes account of the curvature, (as an information variable say) inflation will be closer to the target and similarly output will be closer to trend. This means that under optimal policy the observed (reduced form) Phillips curve will almost certainly be either

the expected sum of discounted squared future deviations from the inflation target. This is consistent with the UK's New Monetary Framework, where the operational target for monetary policy is an underlying inflation rate (measured by the 12-month increase in the RPI excluding mortgage interest payments) of 2.5 per cent. For simplicity we focus on the inflation objective and abstract from output stabilisation¹¹ and monitoring issues.¹²

Following Bean (1996), it is convenient to formulate this optimisation problem using dynamic programming. Let $V(\mathbf{p}_t)$ be the minimised expected present value in (2.6) (the value function). Then:

$$V(\mathbf{p}_t) = \underset{\{i_t\}}{\text{Min}} \left\{ E_t \left[\frac{(\mathbf{p}_t - \mathbf{p}^*)^2}{2} + dE_t[V(\mathbf{p}_{t+1})] \right] \right\} \quad (2.7)$$

Using (2.1) this can be written as

$$V(\mathbf{p}_t) = \underset{\{i_t\}}{\text{Min}} \left\{ E_t \left[\frac{(\mathbf{p}_t - \mathbf{p}^*)^2}{2} + dE_t[V(\mathbf{p}_t + f(\bullet))] \right] \right\} \quad (2.8)$$

subject to (2.4) and (2.5). Note that if $\mathbf{j} = 0$ we obtain the Svensson (1997a) model exactly.

Since the interest rate affects inflation with a two year lag, it is practical to express \mathbf{p}_{t+2} in terms of year t and $t+1$ variables.

Leading the Phillips curve one period and substituting for output from (2.1) yields

$$\mathbf{p}_{t+2} = \mathbf{p}_t + \frac{\mathbf{a}_1 y_t}{1 - \mathbf{a} \mathbf{j} y_t} + \frac{\mathbf{a}_1 y_{t+1}}{1 - \mathbf{a} \mathbf{j} y_{t+1}} \quad (2.9)$$

As in Svensson (1997a) the interest rate in year t does not affect the inflation rate in year t and $t+1$, but only in year $t+2$, $t+3$ etc, similarly the interest rate in year $t+1$ will only affect the inflation rate in year $t+3$, $t+4$

linear or non-existent. Thus, the more the central bank takes account of possible asymmetric (ex ante) inflation risks because of perceived nonlinearities in the inflation output relation, the less visible they will be in the data as a result. This problem has been studied formally by Laxton, Rose and Tambakis (1997).

¹¹ Svensson (1997a, pp. 1130-1134) shows that the weight on output stabilisation determines how *quickly* the inflation forecast is adjusted towards the inflation target. This is the most realistic case and also relevant for the UK situation. The reason is that it is recognised by the Chancellor that sticking to the inflation target - in the case of external events or temporary difficulties - may cause undesirable volatility in output. However, in the more complicated case of multiplier uncertainty, Svensson (1997b) also focuses on strict inflation targeting. In order to keep our (already fairly complicated) analysis tractable, we focus on strict inflation targeting. Moreover, this facilitates comparison with the Svensson (1997b) results.

¹² Svensson (1997a, p. 1123) states that: "Central banks have a strong tradition of secrecy mostly for no good reasons I believe". For an alternative view where central bank secrecy may be beneficial because of a positive effect on output stabilisation see Eijffinger, Hoerberichts and Schaling (1997).

etc¹³. Therefore we can find the solution to the dynamic programming problem by assigning the interest rate in year t to the inflation target for year $t+2$, the interest rate in year $t+1$ to the inflation target for year $t+3$ etc. Thus, we can find the optimal interest rate in year t as the solution to the simple period by period problem¹⁴

$$\text{Min}_{i_t} E_t d^2 \left[\frac{(p_{t+2} - p^*)^2}{2} \right] \quad (2.10)$$

The first order condition for minimising (2.10) with respect to i_t is

$$\begin{aligned} \frac{\mathbb{J} E_t d^2 L(p_{t+2})}{\mathbb{J} i_t} &= \frac{d^2}{2} [2(E_t p_{t+2} - p^*) \frac{\mathbb{J} E_t p_{t+2}}{\mathbb{J} i_t}] = \\ & - \frac{d^2 a_1}{(1 - \mathbf{a} \mathbf{j} [\mathbf{b}_1 y_t - (i_t - p_t) + \mathbf{b}_2 x_t])^2} (E_t p_{t+2} - p^*) = 0 \end{aligned} \quad (2.11)^{15}$$

where we have used that by (2.9) the effect of interest rate increments on expected inflation two years ahead is

$$\frac{\mathbb{J} E_t p_{t+2}}{\mathbb{J} i_t} = \frac{\mathbb{J} E_t p_{t+2}}{\mathbb{J} E_t y_{t+1}} \cdot \frac{\mathbb{J} E_t y_{t+1}}{\mathbb{J} i_t} = - \frac{a_1}{(1 - \mathbf{a} \mathbf{j} [\mathbf{b}_1 y_t - (i_t - p_t) + \mathbf{b}_2 x_t])^2} = -f'(E_t y_{t+1}) \quad (2.11.a)$$

It follows that the first-order condition can be written as

$$E_t p_{t+2} = p^* \quad (2.12)$$

Hence, as in Svensson (1997a, p. 1118) the interest rate in year t should be set so as that the *inflation forecast* for p_{t+2} , the mean of inflation conditional upon information available in year t , equals the inflation target.

The one-to-two-year inflation forecast is given by

$$E_t p_{t+2} = p_t + f(\bullet) + E_t f(\bullet)_{t+1} \quad (2.13)$$

¹³ This set-up is a stylised version of the Bank of England's forecasting model where three quarters of the effect of interest rate changes on inflation occurs in two year's time.

¹⁴ For a proof see Appendix A of Svensson (1997a).

¹⁵ For analytical tractability in this Section we do not analyse the implications of uncertainty about the output gap. This makes the analysis fairly complicated, as it implies solving a nonlinear stochastic control problem that excludes closed form solutions for interest rates. We analyse this issue in Section 4.

The last term is the forecast of the inflationary pressure as implied by next year's output gap. Using (2.1) and (2.4) this forecast is

$$E_t f(\bullet)_{t+1} = E_t \left(\frac{\mathbf{a}_1 [\mathbf{b}_1 y_t - r_t + x_{t+1}]}{1 - \mathbf{a}_j [\mathbf{b}_1 y_t - r_t + x_{t+1}]} \right) = \frac{\mathbf{a}_1 [\mathbf{b}_1 y_t - r_t + \mathbf{b}_2 x_t]}{1 - \mathbf{a}_j [\mathbf{b}_1 y_t - r_t + \mathbf{b}_2 x_t]} = f(E_t y_{t+1}) \quad (2.14)^{16}$$

where $r = i - \mathbf{p}$ is the real base rate.

Substituting (2.1) and (2.14) into (2.13) and setting the one-to-two year inflation forecast equal to the inflation target leads to the central bank's *optimal policy rule*

$$r = \frac{(1 - \mathbf{a}_j [\mathbf{b}_1 y_t - r + \mathbf{b}_2 x_t])}{\mathbf{a}_1} (\mathbf{p}_t - \mathbf{p}^*) + \frac{(b_1 - 2\mathbf{a}_j \mathbf{b}_1 y_t - \mathbf{a}_j [\mathbf{b}_2 x_t - r_t])}{[1 - \mathbf{a}_j y_t]} y_t + \mathbf{b}_2 x_t \quad (2.15)$$

where $b_1 = (1 + \mathbf{b}_1)$

According to this equation the optimal short-term interest rate is a *nonlinear* function of the deviation from the inflation target ($\mathbf{p} - \mathbf{p}^*$) on the one hand, and the output gap (y), on the other.

This is in contrast to Bean (1996), who gets a *linear* policy rule. This is due to the fact that he employs a specific functional form for the nonlinear Phillips curve.¹⁷

An important limiting case of (2.15) is when \mathbf{j} becomes very small. In the latter case the Phillips curve approaches the standard linear functional form and the policy rule collapses to

$$r - r^* = a_1 (\mathbf{p}_t - \mathbf{p}^*) + b_1 y_t \quad (2.16)^{18}$$

where $a_1 = \frac{1}{\mathbf{a}_1}$, $r^* = \mathbf{b}_2 x_t$

which - as in Svensson (1997, p.1119) - is essentially a *forward looking* version of the simple backward looking reaction function popularised by Taylor (1993). In what follows for brevity's sake I

¹⁶ Because we abstract from the implications of uncertainty about the output gap there is no Jensen's inequality effect in (2.14). We address this extension in Section 4.

¹⁷ In fact his specification is probably the *only* specification that (together with standard quadratic preferences over inflation and output) implies a linear policy rule as the solution to the associated dynamic programming problem.

¹⁸ Note that this solution *does* take account of uncertainty about the output gap. The reason is that because of *certainty equivalence*, the optimal control trajectory for the stochastic problem is identical with the solution to the deterministic problem when the error terms take their (zero) expected values.

will refer to (2.16) as the Taylor rule.¹⁹ The nonlinear rule (2.15) will be analysed in detail in the next Section.

3 A Nonlinear Policy Rule

In this Section we focus on the properties of the nonlinear rule. We show that nominal interest rates according to this rule are higher than under the Svensson (1997a) forward looking version of the Taylor rule. This means that - if the economy is characterised by asymmetries - the Svensson rule may underestimate the correct level of interest rates.

To recap we focus on our initial result, i.e. equation (2.15)

Rearranging and using that $b_2 x_t = r^*$ we get

$$r - r^* = \frac{1/a_1}{\{1 - j[(p - p^*) + f(\bullet)]\}}(p - p^*) + \frac{1/a_1 f(\bullet)}{\{1 - j[(p - p^*) + f(\bullet)]\}} + b_1 y_t \quad (3.1)$$

Equation (3.1) is the central result of this Section and shows that the real interest-rate penalty $r - r^*$ is a nonlinear function of the deviation of the inflation rate from its target $p - p^*$ and the output gap y . In order to make progress it is useful to focus on the inflation argument in the rule. That is, for the moment we set $y = 0$ in (3.1). This yields

$$r - r^* = \frac{1/a_1}{\{1 - j(p - p^*)\}}(p - p^*) \quad (3.1.a)$$

The most interesting feature of (3.1.a) is that the elasticity of the interest rate penalty with respect to deviations from the inflation target is *state-contingent*. Meaning that this elasticity depends on the *level of inflation*.

To give a numerical example, consider the effects of a + 0.5% and a - 0.5 % deviation of inflation from target. We analyse the implications of these inflation gaps for short-term interest rates under the following parameter values $a_1 = 0.5$, $j = 0.5$ and $r^* = 3.80$. Then, the appropriate interest rate penalties are +1.33 and -0.80% respectively. In the linear case (Taylor rule) we get +1.00 and -1.00%

¹⁹ Also, it should be emphasised that the original Taylor rule is an *instrument rule*: it directly specifies the reaction function for the instrument in terms of current information. In contrast a *target rule* results in an endogenous optimal reaction function expressing the instrument as a function of the available relevant information. For this distinction see Svensson (1997a, p. 1136). We call (2.16) forward-looking because - although interest rates feed off current-dated variables only - the latter are *leading indicators* of future inflation. For more details see Svensson (1997a).

Hence the interest rate response is *asymmetric*; positive deviations from the inflation target imply higher (absolute values of) real interest rate penalties than negative deviations.²⁰

The intuition behind this result is the following. If inflation is *above* target, short-term real interest rates will be *below* their equilibrium level. The result of this is that there are inflationary pressures in the economy that - if left to their own devices - will increase tomorrow's output gap. Since the Phillips curve is nonlinear this positive output gap at time $t + 1$ will increase the inflation rate at time $t + 2$ *by more* than if the world was linear. To offset this the central bank needs to increase nominal interest rates at time t *further* than in the Svensson model. Of course, in case of a negative deviation from the inflation target, the reverse is true. That is, then real interest rates are *above* their equilibrium level. The associated *disinflationary* pressures will depress tomorrow's output gap. However, this will now cause *less* disinflation than in the linear case. Hence, the central bank does not need to cut rates as much.

Now, we focus on the output gap argument; hence we look at the opposite case of the one analysed above. Setting $\mathbf{p} = \mathbf{p}^*$ in (3.1) yields

$$r - r^* = \frac{1 / \mathbf{a}_1 f(\bullet)}{\{1 - \mathbf{j} f(\bullet)\}} + \mathbf{b}_1 y_t \quad (3.1.b)$$

It can be shown that (3.1.b) has characteristics similar to (3.1.a). In particular the elasticity of the interest rate with respect to output depends on the *level of the output gap*. To give a numerical example, consider the effects of a +0.50 % and -0.50 % output gap on the real interest rate penalty. Using the same parameters as in the inflation example, and setting $\mathbf{b}_1 = 0.7$, we get +1.02 % and -0.75% respectively. In the linear case (Taylor rule) we get +0.85% and -0.85% respectively.

Thus, also with respect to the output gap the interest rate response is asymmetric. Positive output gaps imply higher (absolute values of) real interest rate penalties than negative output gaps.²¹ The intuition is similar to the one of the inflation argument in the rule. If output is *above* trend at time t , then because of serial correlation in output, tomorrow's output gap will be higher as well. Then, because of the asymmetry the inflation rate at time $t + 2$ will increase *by more* than if there were no asymmetries. In order to prevent all this from happening, the central bank needs to put up nominal interest rates by more than according to the forward looking version of the Taylor rule. Similarly, in

²⁰ Note that applying Svensson's distinction between 'official' versus *implicit* inflation targets - and for ease of exposition setting $y = 0$ - it is possible to reformulate the nonlinear policy rule (3.1.a) as a *linear* response to a nonlinear (state-contingent) implicit inflation target \mathbf{p}_t^b . After some algebra it can be shown that then (2.16) can be

reformulated as $r - r^* = a_1(\mathbf{p}_t - \mathbf{p}_t^b)$ where $\mathbf{p}_t^b \equiv \frac{\mathbf{p}^* - \mathbf{j} \mathbf{p}(\mathbf{p} - \mathbf{p}^*)}{1 - \mathbf{j}(\mathbf{p} - \mathbf{p}^*)}$.

²¹ Clarida and Gertler (1997a) have found that it is possible to represent Bundesbank policy actions in terms of an interest rate reaction function which maps back into a Taylor-type rule. Their specification allows a modified Taylor rule with *linear* responses to expected inflation and *asymmetric* responses to the output gap.

case of a negative output gap the danger of *disinflation* is less severe, calling for a less substantial cut than according to the linear rule.

The above analysis sheds some light on the mechanics of our policy rule (3.1). However, this was done by focusing on the inflation ‘gap’, given a zero output gap and vice versa. In the real world it is not very likely that those are the only relevant cases. Therefore now we drop this restriction and allow both gaps to vary simultaneously. To get a feel for what happens then consider Table 3.1.

Table 3.1 Implications of Policy Rules for Short Term Interest Rates²²

| Inflation minus Target | Output Gap | Real Rate Penalty | Idem ‘Taylor’ Rule | Nom Interest Rate ‘Taylor’ Rule (2.16) ²³ | Idem Non Linear Rule (3.1) | Idem with Unc About Output Gap (4.4) | Interest Rate Bias in Basis Points ²⁴ | ‘Brainard’ Effect in Basis Points |
|------------------------|------------|-------------------|--------------------|--|----------------------------|--------------------------------------|--|-----------------------------------|
| -0.50 | -0.50 | -1.41 | -1.85 | 3.95 | 4.39 | 4.80 | 44 (41) | + 30 |
| -0.50 | 0.00 | -0.80 | -1.00 | 4.80 | 5.00 | 5.34 | 20 (34) | + 23 |
| 0.00 | -0.50 | -0.75 | -0.85 | 5.45 | 5.55 | 5.80 | 10 (25) | + 15 |
| 0.00 | 0.00 | 0.00 | 0.00 | 6.30 | 6.30 | 6.50 | 0 (20) | + 10 |
| -0.50 | 0.50 | -0.04 | -0.15 | 5.65 | 5.76 | 6.02 | 11 (25) | + 15 |
| 0.50 | -0.50 | 0.30 | 0.15 | 6.95 | 7.10 | 7.24 | 15 (14) | + 5 |
| 0.00 | 0.50 | 1.02 | 0.85 | 7.15 | 7.32 | 7.46 | 17 (14) | + 5 |
| 0.50 | 0.00 | 1.33 | 1.00 | 7.80 | 8.13 | 8.24 | 33 (11) | + 3 |
| 0.50 | 0.50 | 2.94 | 1.85 | 8.65 | 9.74 | 9.82 | 109 (8) | + 1 |

This Table maps output and inflation gaps into real interest rate *penalties* (columns 3 and 4), and into *nominal* interest rates (the shaded columns 5, 6 and 7). Please note that this Table is *not* computed by stochastic simulations. All that is necessary to obtain the numbers in this Table is to start with certain output - and inflation gaps, and plug these into the policy rule (3.1) (and (4.4) for column 7), given the parameter values used earlier. Also note that our previous numerical examples are reported in rows 8 and 2, (for the inflation example) and rows 7 and 3 (for the output gap example).

²² Note that while Taylor prescribes coefficients of one half on both the inflation and output gaps under plausible parameter values the ‘Svensson’ rule responds to inflation and output gaps with elasticities of 2 and 1.7 respectively. In this respect see Broadbent (1996), who finds numbers of 5 and 3.5. Also, as pointed out by Svensson (1997a, p. 1133) with a positive weight on output stabilisation, the coefficients in the optimal reaction function - and consequently the numbers in the Table - will be smaller.

²³ Nominal interest rate = $r + p = (r - r^*) + r^* + p$, where $p = (p - p^*) + p^*$.

²⁴ First number = (3.1) -/-(2.16). Bias due to uncertainty = (4.4) -/-(3.1) in brackets. The effects of uncertainty will be explained in Section 4.

Consider first the shaded row. This row corresponds with the case of *neutral monetary conditions*. Meaning that the economy is operating at full potential (zero output gap) and inflation on course (equal to the inflation target). Thus both gaps are zero and real interest rates are at their equilibrium level. Note that in this case the linear and nonlinear policy rules imply the same level of short-term interest rates.

However, by looking at the other rows in this Table it becomes immediately clear that in all other cases short-term interest rates are always higher under the nonlinear rule.

To see this consider the first set of numbers in column 8. The difference in nominal rates is zero for neutral monetary conditions but ranges from about 40 to 100 basis points otherwise. Hence, the numbers suggest that interest rates are higher in a nonlinear than in a linear world.

In order to investigate this conjecture formally consider equation (3.2)

$$r_{NL} - r_L = \frac{1 / \mathbf{a}_1[(\mathbf{p} - \mathbf{p}^*) + f(\bullet)]}{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet)]} - 1 / \mathbf{a}_1(\mathbf{p} - \mathbf{p}^*) - y_t \quad (3.2)$$

This is the algebraic equivalent of the first set of numbers in column 8 of Table 3.1. It is obtained by subtracting the level of interest rates according to the Taylor rule r_L (given by equation (2.16)) from that under the nonlinear rule r_{NL} (equation (3.1)).

From equation (3.2) we conclude that the level of short term interest rates as implied by the nonlinear policy rules is *higher* than under the Taylor rule. For a proof see the Appendix A, where we show that (3.2) has a *local minimum* at $(\mathbf{p} - \mathbf{p}^*, y) = (0,0)$. Hence, under non-neutral monetary conditions interest rates according to the nonlinear rule are higher than under the Taylor rule.

The intuition is the following. If the Phillips curve is nonlinear, then positive shocks to demand - in the form of positive output and/or inflation gaps- are more dangerous for inflation than if the world is symmetric. This means that the central bank will need to raise rates by more than in the Svensson model. Similarly, negative gaps will be less disinflationary urging the central bank to cut by less. Of course the net result is that nominal interest rates are higher on average.

Note that there is one interesting intermediate case which we did not investigate.²⁵ This is the scenario where the model is *nonlinear*, but the policy rule remains *linear* (i.e. of the form given by (2.16)).

Using stochastic simulations it can then be shown that interest rates will be higher than under a *linear* Phillips curve with a *linear* policy rule. Moreover, it can then be analysed how much further interest rates need to rise under the *optimal* (asymmetric) policy rule compared to the linear rule. The level of interest rates in the nonlinear model under the nonlinear policy rule can then be decomposed into two parts: (i) the jump in rates caused by the change from a linear to a nonlinear model (where the policy rule remains linear), and (ii) the further change in rates (in the nonlinear model) caused by the switch

²⁵ I owe this suggestion to Peter Westaway.

from a linear to a nonlinear policy rule. The stochastic simulations show that both the effects under (i) and (ii) are positive, where the effect under (i) is quantitatively the most important.²⁶

4. Uncertainty about the Output Gap

Now we analyse the effects of uncertainty about the output gap on the setting of short-term interest rates. This means that we now analyse random shocks to the output gap. This effect is captured in the model by the term \mathbf{e}_{t+1} in equation (2.5). Thus, from the perspective of the central bank, the inflation rate becomes a random variable that can only be *imperfectly* controlled.²⁷ More specific, because of the nonlinearity of the economy, uncertainty about the true value of next year's output gap implies that the *slope* of the Phillips curve - and hence the *effect* of interest rate increments on inflation two years ahead - also becomes a random variable. Hence, the combination of *additive* uncertainty about the economy combined with a nonlinear structure gives rise to issues of *multiplier* or *model* uncertainty. However, here the implications for optimal policy are quite different from either the standard Brainard (1967) analysis, or Svensson's (1997b) extension of his inflation forecast targeting framework with model uncertainty.

We now extend the analysis of Section 3. As stated in Section 2 we can find the optimal base rate in year t as the solution to the problem

$$\underset{i_t}{\text{Min}} E_t d^2 \left[\frac{(\mathbf{p}_{t+2} - \mathbf{p}^*)^2}{2} \right] = \underset{i_t}{\text{Min}} E_t d^2 L(\mathbf{p}_{t+2}) \quad (2.10)$$

subject to (2.1), (2.4) and (2.5)

We can rewrite the expected value of the discounted loss as²⁸

$$\begin{aligned} E_t d^2 L(\mathbf{p}_{t+2}) &= E_t d^2 \left[\frac{(\mathbf{p}_{t+2} - E_t \mathbf{p}_{t+2}) + (E_t \mathbf{p}_{t+2} - \mathbf{p}^*)}{2} \right]^2 = \\ &= \frac{d^2}{2} [Var_t \mathbf{p}_{t+2} + (E_t \mathbf{p}_{t+2} - \mathbf{p}^*)^2] \end{aligned} \quad (4.1.a)$$

and we can define

²⁶ The results are available from the author upon request.

²⁷ This is also true in the linear stochastic model but there the forecast error does not depend on the interest rate.

²⁸ Using $\mathbf{p}_{t+2} = E_t \mathbf{p}_{t+2} + (\mathbf{p}_{t+2} - E_t \mathbf{p}_{t+2})$.

$E_t \mathbf{p}_{t+2} \equiv \overline{\mathbf{p}_{t+2}} + (E_t \mathbf{p}_{t+2} - \overline{\mathbf{p}_{t+2}}) = \overline{\mathbf{p}_{t+2}} + E_t d_{t+2}$, i.e. the one to two year inflation forecast equals the *deterministic* (or certainty equivalent) inflation forecast

$$\overline{\mathbf{p}_{t+2}} = \mathbf{p}_t + f(\bullet) + f(E_t y_{t+1}) \quad (4.1.b)$$

where $f(E_t y_{t+1}) = \frac{\mathbf{a}_1 [\mathbf{b}_1 y_t - r_t + \mathbf{b}_2 x_t]}{1 - \mathbf{a}_j [\mathbf{b}_1 y_t - r_t + \mathbf{b}_2 x_t]}$

plus the *expected deviation* $E_t d_{t+2}$ of the one to two year inflation forecast from the certainty equivalent forecast:

$$\begin{aligned} E_t d_{t+2} &= E_t \mathbf{p}_{t+2} - \overline{\mathbf{p}_{t+2}} = (\mathbf{p}_t + f(\bullet) + E_t f(\bullet)_{t+1}) - (\mathbf{p}_t + f(\bullet) + f(E_t y_{t+1})) \\ &= E_t f(\bullet)_{t+1} - f(E_t y_{t+1}) \end{aligned} \quad (4.1.c)$$

This split is important because it will enable us to identify one of the two channels through which the uncertainty affects *inflation forecast targeting*.

Substituting the decomposition of the one to two year inflation forecast into (4.1.a) gives

$$\begin{aligned} E_t d^2 L(\mathbf{p}_{t+2}) &= \frac{d^2}{2} [Var_t \mathbf{p}_{t+2} + (\overline{\mathbf{p}_{t+2}} + E_t d_{t+2})^2 \\ &+ (\mathbf{p}^*)^2 - 2\mathbf{p}^* (\overline{\mathbf{p}_{t+2}} + E_t d_{t+2})] \end{aligned} \quad (4.2)$$

The advantage of (4.2) over (2.10) is that the stochastic elements of the solution have been isolated in the terms $E_t d_{t+2}$ and $Var_t \mathbf{p}_{t+2}$. It is precisely through these two terms that the uncertainty about the output gap affects inflation forecast targeting.

We will now derive the policy rule in the presence of both asymmetries and uncertainty. Because the rule is highly nonlinear, unlike the previous Section it is not possible to derive an *explicit* function that maps output and inflation gaps into the appropriate level of interest rates. Instead we resort to numerical methods. However, we are able to derive robust qualitative analytical results.

The punch line is that no matter what parameter values, nominal interest rates will be *higher* the higher the uncertainty about the output gap

The first order condition is

$$\overline{(\mathbf{p}_{t+2} - \mathbf{p}^*)} + E_t d_{t+2} + \frac{\mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t}{2[\mathbb{J}\mathbf{p}_{t+2} / \mathbb{J}i_t + \mathbb{J}E_t d_{t+2} / \mathbb{J}i_t]} = 0 \quad (4.3)^{29}$$

Where the first term is the difference between the certainty equivalent inflation forecast (4.1.b) and the inflation target, the second term is the expected deviation of the one to two year inflation forecast from its certainty equivalent value (4.1.c), and the last term captures the effect of nominal interest rates on the conditional variance of inflation, that is on the *variability* or 'risks' surrounding the central forecast.

Substituting (4.1.b) and (4.1.c) into (4.3) and rearranging leads to the central bank's *optimal policy rule*

$$\begin{aligned} r - r^* = & \frac{1/a_1}{\{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet) + E_t d_{t+2} + \frac{\mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t}{2[\mathbb{J}\mathbf{p}_{t+2} / \mathbb{J}i_t + \mathbb{J}E_t d_{t+2} / \mathbb{J}i_t]}]\}} (\mathbf{p} - \mathbf{p}^*) \\ & + \frac{1/a_1 f(\bullet)}{\{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet) + E_t d_{t+2} + \frac{\mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t}{2[\mathbb{J}\mathbf{p}_{t+2} / \mathbb{J}i_t + \mathbb{J}E_t d_{t+2} / \mathbb{J}i_t]}]\}} \\ & + \frac{1/a_1 (E_t d_{t+2} + \frac{\mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t}{2[\mathbb{J}\mathbf{p}_{t+2} / \mathbb{J}i_t + \mathbb{J}E_t d_{t+2} / \mathbb{J}i_t]})}{\{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet) + E_t d_{t+2} + \frac{\mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t}{2[\mathbb{J}\mathbf{p}_{t+2} / \mathbb{J}i_t + \mathbb{J}E_t d_{t+2} / \mathbb{J}i_t]}]\}} + \mathbf{b}_1 y_t \end{aligned} \quad (4.4)$$

where

$$\begin{aligned} E_t d_{t+2} &= (\mathbf{j} / 2) f''(E_t y_{t+1}) \mathbf{s}_e^2 > 0 \\ \mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t &= -2 f'(E_t y_{t+1}) f''(E_t y_{t+1}) \mathbf{s}_e^2 < 0 \\ \overline{\mathbb{J}\mathbf{p}_{t+2}} / \mathbb{J}i_t &= -f'(E_t y_{t+1}) < 0 \\ \mathbb{J}E_t d_{t+2} / \mathbb{J}i_t &= -(\mathbf{j} / 2) f'''(E_t y_{t+1}) \mathbf{s}_e^2 < 0 \end{aligned} \quad (4.5)$$

According to equation (4.4) the optimal short-term interest rate is determined by the deviation from the inflation target $(\mathbf{p} - \mathbf{p}^*)$ on the one hand and the output gap y (through the terms $\mathbf{b}_1 y_t$ and $f(\bullet)$) on the other.

²⁹ Note that in the *deterministic* case $E_t d_{t+2} = \mathbb{J}Var_{\mathbf{p}_{t+2}} / \mathbb{J}i_t = 0$ and we get $\overline{\mathbf{p}_{t+2}} = \mathbf{p}^*$ which is the first order condition in the *certainty equivalence case* as in Svensson (1997a, p. 1118).

An important limiting case of (4.4) is when s_e^2 becomes very small. In the latter case the stochastic elements of the rule, $E_t d_{t+2}$, $\mathbb{E} E_t d_{t+2} / \mathbb{E} i_t$ and $\mathbb{E} \text{Var} p_{t+2} / \mathbb{E} i_t$ become very small as well, and the policy rule collapses to

$$\frac{1/a_1}{\{1 - j[(p^*)]\}}(p) = \frac{1/a_1(\cdot)}{\{1 - j[(p^*)]\}} b_t, \quad (3.1)$$

which is the asymmetric policy rule for the case where $j > 0$, that is the *certainty equivalent rule* the nonlinear model. Of course, if we set $j = 0$ in (3.1), the asymmetric certainty equivalent rule collapses to the

$$(2.16)$$

Svensson result. Table 4.1 summarises the cases discussed above.

Table 4.1 Classification of Policy Rules

| | Uncertainty about the Output Gap | |
|----------------------|--|----------------------------|
| | No Uncertainty $s_e^2 = 0$ | Uncertainty $s_e^2 > 0$ |
| Linear $j = 0$ | Svensson Result (2.16) | Svensson Result (2.16) |
| Nonlinear $j > 0$ | Nonlinear Certainty Equivalent Rule (3.1) | Nonlinear Rule (4.4) |

Turning to the case where both j and s_e^2 are positive, from equations (4.4) and (4.5) it can be seen that the stochastic elements of the rule $E_t d_{t+2}$, $\mathbb{E} E_t p_{t+2} / \mathbb{E} i_t$ and $\mathbb{E} \text{Var} p_{t+2} / \mathbb{E} i_t$ depend on the level of the interest rate. Thus, *both* the left-hand side and the right-hand side of equation (4.4) depend on the interest rate. Therefore, it is not possible to derive an *explicit* function that maps output and inflation gaps into the appropriate level of interest rates. Instead, we have to resort to numerical methods to find the level of the real interest rate that is implicitly determined by equation (4.4).

Setting s_e^2 at 0.925³⁰ and keeping the the real interest rate at the *certainty equivalent* level according to rule (3.1) we can compute the effect of the uncertainty on the inflation forecast and on the *risks* surrounding the forecast.

³⁰ Being the MSE of ONS revisions to real GDP in the late 1980s. For more details see Dicks (1997). Obviously this is a crude way of parametrising the model, but in the linear case there is a one to one correspondence between the conditional variance of the output gap at time t and the variance of shocks s_e^2 . Also, this highlights another attractive feature of the model. We have a natural mapping of noisy data (which is very much a real life problem) into issues of multiplier uncertainty.

We find that the inflation forecast is adjusted upwards. This forecast now *overshoots* the 2.5% target level that would be attained in two year's time with interest rates according to (3.1) and no uncertainty. Moreover, the same is true for the conditional variance of inflation. At the level of interest rates implied by the certainty equivalent rule (3.1) we get a variance that goes up to 86% of the variance of the shock to the output gap. This means that only a very small amount of the demand shock is dampened before it passes through and causes significant inflation risks. Clearly in the presence of uncertainty interest rates according to (3.1) are at a suboptimal level.

To find the appropriate level we numerically compute the real interest rate that solves the first order condition. The results can be found in column 7 of Table 3.1.

It becomes immediately clear that short-term interest rates according to rule (4.4) are *higher* than under the certainty equivalent nonlinear rule (3.1). To see this consider the numbers in brackets in column 9. The difference due to the uncertainty is about 25 bp for neutral monetary conditions and ranges from about 10 to 40 bp otherwise.³¹ This means that uncertainty induces a *further* upward bias in nominal interest rates on top of the effect of the nonlinearity per se as analysed in Section 3.

In order to investigate these results more formally consider (4.4). In this equation the stochastic elements of the solution have been isolated in the terms $E_t d_{t+2}$, $\mathbb{J}E_t d_{t+2} / \mathbb{J}i_t$ and $\mathbb{J}Var_t \mathbf{p}_{t+2} / \mathbb{J}i_t$.

Now, the sign of $E_t d_{t+2}$ in (4.1.c) will always be positive, implying that the one to two year inflation forecast will be *higher* than the certainty equivalent inflation forecast as derived in Section 3. The reason is that positive shocks to the output gap are *more inflationary* than negative shocks are *disinflationary*, hence with equal probabilities of positive and negative shocks, the inflation forecast will be adjusted upwards, and the more so the higher the variance of shocks hitting the output gap \mathbf{s}_e^2 . This can be restated in a more technical way by noting that the forecast of tomorrow's inflationary pressures, $E_t f(\bullet)_{t+1}$, involves the expectation of a convex function which will always be higher than the value of the f function at the expected value, $f(E_t y_{t+1})$. Hence, the first channel through which the uncertainty affects inflation forecast targeting is the *Jensen's inequality effect*. Note that from (4.5) this effect becomes smaller the higher the interest rate, i.e. $\mathbb{J}E_t \mathbf{p}_{t+2} / \mathbb{J}i_t < 0$.

The *second* channel through which the uncertainty affects inflation forecast targeting is through its effects on the *conditional variance* of the one to two year inflation forecast $Var_t \mathbf{p}_{t+2}$. This is important because it implies that in case of imperfect control of the inflation rate the policymaker should also take account of the *risks* surrounding the central inflation projection.

³¹ Note that strictly speaking the definition of neutral monetary conditions needs to be changed in the nonlinear model. The reason is that now the natural rate of output lies below the natural rate of output in the linear model. With the parameter values in the paper this difference amounts to about -0.1% of GDP. Therefore neutral monetary conditions now means inflation at target and output at the adjusted natural rate. Indeed it can be shown that with inflation on target and output at -0.13 the interest rate bias disappears and the appropriate level of the real interest rate (as defined by the policy rule (4.4) is equal to $r^* = 3.8$.

It can be shown that this variance is

$$\text{Var}_t \mathbf{p}_{t+2} = [f'(E_t y_{t+1})]^2 \mathbf{s}_e^2 \quad (4.6)$$

Now, from (4.5) it can be seen that by increasing interest rates this variance can be reduced. The reason is that by putting up rates, today's forecast of tomorrow's output gap goes down. This means that next year's Phillips curve will be *flatter* which in turn implies that the effects of demand shocks at time $t+1$ on inflation in two year's time will be smaller. Hence, the variability of inflation around the central projection can be reduced by increasing short-term interest rates. For instance, returning to our earlier numerical example, by putting up rates to their appropriate level the conditional variance of inflation is reduced from 86% to about 51% of the initial variance of demand shocks.

The implication for policy is that with uncertainty about the output gap (and asymmetries in the output inflation trade-off), cautious policymaking implies a more activist (more aggressive) rather than a less activist (more passive) interest rate policy.

To recap, the intuition is that a higher variance of shocks hitting the output gap implies a higher inflation forecast (through the Jensen's inequality effect) and a higher conditional variance of inflation. Both can be reduced by increasing nominal interest rates above their certainty equivalent level.

To see the benefits of this policy from a different perspective, consider the implications of stabilisation for the *level* of output. With a convex Phillips curve, the mean level of output is inversely related to the *variability* of inflation around the central projection. Therefore a monetary strategy that reduce this variability (by responding correctly to the multiplier uncertainty issue) does not only keep the inflation rate closer to the target, but also has the important added bonus of pushing up the level of output.³²

5. Brainard Uncertainty and Nonlinearities

Note that the results with respect to the conditional variance of inflation are the *opposite* of those of the standard Brainard (1967) multiplier uncertainty analysis.³³ There uncertainty about the effects of policy calls for a *less activist* policy. The reason is that - according to Brainard's analysis - the variance of the target variable is a linear function of the variance of the policy multiplier. Moreover, the latter is positively related to the level of the instrument. It follows that policies that are 'too

³² I owe this insight to Clark et al (1995, p. 8). They in turn quote Mankiw (1988, p. 483). The result can be verified by inverting the Phillips curve (2.1). This yields $y_t = \frac{\Delta \mathbf{p}_{t+1}}{\mathbf{a}_1(1 + \mathbf{j} \Delta \mathbf{p}_{t+1})}$. Now leading this equation one period and taking expectations at time t of the resulting *concave* function yields that mean output, $E_t y_{t+1}$, is inversely related to the conditional variance of inflation $\text{Var}_t \mathbf{p}_{t+2}$.

activist' increase the variance of the target variable thereby deteriorating the performance of stabilisation policy.

In this Section we show that in the nonlinear stochastic model uncertainty about the effects of policy calls for more a *more activist* policy. Thus, the Brainard result is reversed.

In his (1967) paper Brainard identified two types of uncertainty that may face a policymaker. First, at the time he must make a policy decision he is uncertain about the impact of the exogenous variables which affect the goal variable. This may reflect the policymaker's inability to forecast perfectly either the value of exogenous variables or the response of the goal variable to them. Second, the policymaker is uncertain about the response of the goal variable to any given policy action. He may have a central estimate of the expected value of the response coefficient, but he is aware that the actual response of the goal variable to policy action may differ substantially from the expected value.

Let us now rephrase the above in the context of inflation forecast targeting. To make things comparable with Brainard, for the moment we focus on the *linear* version ($\mathbf{j} = 0$) of the stochastic model presented earlier. Type 1 uncertainty means that when the central bank sets its instrument variable, the nominal interest rate at time t , it is uncertain about the realisation of the exogenous shock to the output gap at time $t + 1$. Here the central bank's inability to forecast next year's output gap perfectly, implies that it is also unable to forecast inflation perfectly. As a consequence inflation in two year's time will differ from its forecast at time t (which is the basis for its interest rate policy). More specific, if the output gap is higher than expected inflation overshoots its target and vice versa. The second type of uncertainty means that the central bank may have a central estimate of the expected value of the response coefficient of inflation in two year's time with respect to the nominal interest rate at time t , but that it is aware that this central estimate is subject to error. More specific, assume that the central estimate is $-\mathbf{a}_1$ - being the product of the interest elasticity of output (which is -1) and the slope of the Phillips curve (which is \mathbf{a}_1) in the linear stochastic model - and that the variance of this central estimate is \mathbf{s}_t^2 .

Brainard shows that both types of uncertainty imply that the policymaker cannot guarantee that the target variable will assume its target value. But they have quite different implications for policy action. The first type of uncertainty, if present by itself has nothing to do with the actions of the policymaker; it is - as Brainard (1967, p. 413) describes it - 'in the system' *independent* of any action he takes. He then states that if all of the uncertainties are of this type, optimal policy behaviour is *certainty equivalence* behaviour. That is, the policymaker should act on the basis of expected values as if he were certain they would actually occur. Moreover, since in this case the variance and higher moments of the distribution of the goal variable do not depend on the policy action taken, the policymaker's

³³ Throughout the paper if we refer to the Brainard result, we mean Brainard's result for the one instrument and one

actions only shift the location of the target variable's distribution. In the presence of the second type of uncertainty, however, the shape as well as the location of the distribution of the target variable depends on the policy action. In this case the policymaker should take into account his influence on the *variability* of the target variable. In his analysis³⁴ Brainard assumes that the variance of the target variable is a linear and increasing function of the level of the policy instrument. It follows that policies that are 'too activist' increase the variance of the target variable thereby worsening the performance of economic policy. Brainard thus shows that uncertainty about the response coefficient, that is about the policy *multiplier*, leads to an optimal policy that is less active. As the variance of the multiplier rises, the policy of trying to minimise the variance of the target variable tends towards lowering the optimal amount of policy.

Let us now rephrase the above in the context of inflation forecast targeting. An example of *certainty equivalence behaviour* is the Svensson (2.16) forward looking policy rule. This rule is optimal in the *linear* stochastic model. Because shocks to the output gap have a zero expected value at time t , it is optimal for the central bank to act as if these zero values would actually occur.

An example of *uncertainty about response coefficients* is Svensson's (1997b) extension of his inflation forecast targeting framework with multiplier uncertainty. Indeed he finds that multiplier uncertainty calls for a more gradual adjustment of the conditional inflation forecast toward the inflation target. This means that - similar to Brainard - optimal monetary policy will be less activist in the sense that the response coefficients in the optimal policy rule for short-term interest rates decline with the uncertainty.³⁵

target case where the random response coefficient is uncorrelated with the exogenous disturbances.

³⁴ Here we focus on Brainard's most simple case; that is the one instrument and one target case where the random response coefficient is uncorrelated with the exogenous disturbances. The reason for doing this is that this case has the closest correspondence to inflation forecast targeting. There we also have one target, inflation, and one instrument, the nominal interest rate.

³⁵ The above can be derived by resorting to the linear model (setting \mathbf{j} equal to zero) and modifying equation (2.4) as

$$y_{t+1} = \mathbf{b}_1 y_t - \mathbf{t}_t (i_t - \mathbf{p}_t) + x_{t+1} \text{ with } E_t(\mathbf{t}_1) = 1 \quad E_t(\mathbf{t}_1^2) = \mathbf{s}_t^2 \text{ and } E_t(\mathbf{t}_1 \mathbf{e}) = 0 \quad (2.4')$$

This means that now the effects of interest rate changes on tomorrow's output gap are uncertain because the interest elasticity of output is a random variable. If $\mathbf{s}_t^2 \rightarrow 0$ the central estimate is not subject to error and the equation reduces to (2.4). Now it can be shown that $\text{Var}_t \mathbf{p}_{t+2} = \mathbf{a}_1^2 (r^2 \mathbf{s}_t^2 + \mathbf{s}_e^2)$ so that $\mathcal{V} \text{Var}_t \mathbf{p}_{t+2} / \mathcal{V} i_t > 0$ and we obtain the standard Brainard (1967) result. It can be shown that the optimal (linear) policy rule then becomes

$$r = \frac{r^*}{(1 + \mathbf{s}_t^2)} + \frac{a_1}{(1 + \mathbf{s}_t^2)} (\mathbf{p}_t - \mathbf{p}^*) + \frac{b_1}{(1 + \mathbf{s}_t^2)} y_t \quad (2.16')$$

So as in Svensson (1997b) the response coefficients decline with the uncertainty, calling for more cautious policy making. If $\mathbf{s}_t^2 \rightarrow 0$ this rule reduces to (2.16).

Let us now focus on inflation forecast targeting in the *nonlinear* model and relate the effects of uncertainty about the output gap to the Brainard paper. Here - following Brainard's terminology - it appears that we only have type 1 uncertainty. That is, because of an *additive* (white noise) shock to tomorrow's output gap the central bank is unable to forecast inflation perfectly. If the model were linear, certainty equivalence would hold and that would be the end of the story. However in a nonlinear model this uncertainty has very different implications.

Similar to the linear model the uncertainty enters the story through additive shocks to the output gap at time $t + 1$. Suppose now that the output gap is higher than expected. In the nonlinear model the slope of the Phillips curve, $\partial \Delta p_{t+1} / \partial y_t$, depends on the level of the output gap. Because the Phillips curve is convex its *slope* is increasing in the *level* of the output gap (see equation (2.1) and Figure 2.1). Thus, if the output gap turns out to be higher than expected (because of a positive shock), the slope of the Phillips curve is also higher than expected. Similarly, if we have a negative shock the slope of the Phillips curve will be lower than expected.

Interestingly, the above implies that the central bank becomes uncertain about the response of inflation to any given policy action. This response coefficient is equal to the product of the interest elasticity of output (which is -1) and the slope of the Phillips curve (which now depends on the realisation of the additive shock to output).³⁶ Thus if the slope of the Phillips curve is *higher* than expected (because of a positive realisation of the demand shock), the response coefficient of inflation in two year's time with respect to the nominal interest rate at time t is *lower* than expected. Because the response coefficient is negative (increasing nominal interest rates reduces inflation), in this case monetary policy turns out to be *more effective* than expected. Similarly, if the slope of the Phillips curve is lower than expected the response coefficient is higher (less negative) than expected. In this case monetary policy is less effective than expected. To conclude, in the nonlinear model *additive* shocks to the output gap generate uncertainty about the policy *multiplier*; that is type 1 uncertainty has type 2 implications.

From the previous paragraph we learnt that the slope of the Phillips curve depends on the realisation of the shock to the output gap. With a positive realisation monetary policy was shown to be more effective than expected at time t and vice versa. Meaning that the dampening effect of a *given* nominal interest rate at time t , on inflation in two year's time is proportional to the realisation of the shock. However, in this paper we are concerned with *optimal* policy and it is therefore of some interest to relax the assumption of a given nominal interest rate.

Suppose the central bank decides to increase the nominal interest rate. From equation (2.4) it follows that a higher nominal interest rate - *ceteris paribus* - lowers the level of tomorrow's output gap. Moreover, in the nonlinear model the *slope* of the Phillips curve is increasing in the level of the output

³⁶ This can be seen by adapting (2.11.a). The algebraic expression for the response coefficient of inflation in two year's time with respect to the nominal interest rate at time t is

$$\frac{\partial p_{t+2}}{\partial i_t} = \frac{\partial y_{t+1}}{\partial i_t} \cdot \frac{\partial p_{t+2}}{\partial y_{t+1}} = -1 \cdot f'(y_{t+1}) = -\frac{\mathbf{a}_1}{(1 - \mathbf{a}_j [\mathbf{b}_1 y_t - (i_t - p_t) + \mathbf{b}_2 x_t + \mathbf{e}_{t+1}])^2}$$

gap. Thus, a higher nominal interest rate lowers the slope of the Phillips curve. This in turn implies that any positive output shock that may hit the economy at time $t + 1$ will be less inflationary. Similarly, by lowering the slope of the Phillips curve, a higher interest rate will also dampen the disinflationary effects of negative shocks. Thus, in the nonlinear model a higher nominal interest rate causes positive demand shocks to induce *less inflation* and negative shocks to cause *less disinflation*. Of course, if the central bank decides to cut the nominal interest rate, the reverse applies. By increasing the slope of the Phillips curve, a lower interest rate amplifies the inflationary effects of positive output shocks, and enhances the disinflationary effects of negative shocks. Thus, nominal interest rates can dampen or amplify the second round effects of output shocks on inflation. To be more precise, it can be shown that the *conditional variance* of the one to two year inflation forecast, $Var_t \mathbf{p}_{t+2}$, is a *decreasing* function of the nominal interest rate. This can be seen from equations (4.5) and (4.6). As explained above, the reason is that by putting up rates, today's forecast of tomorrow's output gap goes down. This means that next year's Phillips curve will be *flatter* which in turn implies that the effects of demand shocks at time $t+1$ on inflation in two year's time will be smaller. Hence, the variability of inflation around the central projection can be reduced by increasing short-term interest rates.

At this stage it is useful to summarise results so far. First, we have shown that in the nonlinear model *additive* shocks to the output gap imply *uncertainty about policy*; that is type 1 uncertainty has type 2 implications. Second, in the nonlinear model the variance of the target variable (inflation) is a *decreasing* function of the level of the policy instrument (the nominal interest rate). Please note that the second result is the *opposite* of Brainard's (1967) analysis. Brainard assumes that the variance of the target variable is a linear and *increasing* function of the level of the policy instrument. It follows that policies that are 'too activist' increase the variance of the target variable thereby worsening the performance of economic policy.

In contrast here the variance of the target variable is a nonlinear and *decreasing* function of the level of the policy instrument (this can be seen from equations (4.5) and (4.6)). It follows that policies that are 'too activist' from a Brainard perspective may actually *decrease* the variance of the target variable, thereby *improving* the performance of policy. Thus, in the nonlinear model uncertainty about the policy multiplier leads to an optimal policy that is more active. To be more precise, as the variance of the multiplier rises, the policy of trying to minimise the variance of the target variable tends towards *increasing* the optimal amount of policy. This means that the Brainard result is reversed.

To see this we focus on the central bank's optimal policy rule (4.4). As stated before the stochastic elements of this rule are isolated in the terms $E_t d_{t+2}$, $\mathbb{J} E_t \mathbf{p}_{t+2} / \mathbb{J} i_t$ and $\mathbb{J} Var_t \mathbf{p}_{t+2} / \mathbb{J} i_t$. Here the first two terms relate to the effects of the uncertainty on the inflation forecast, and hence capture the *Jensen's inequality effect*. The second channel through which the uncertainty affects inflation forecast

targeting is through its effects on the conditional variance of the one to two year inflation forecast $Var_i \mathbf{p}_{t+2}$.

Now we can isolate the implications of the second channel for the amount of optimal policy by abstracting from the Jensen's inequality effect. This can be done by setting $E_t d_{t+2}$ and $\mathcal{J}E_t \mathbf{p}_{t+2} / \mathcal{J}i_t$ equal to zero in the central bank's optimal rule (4.4). This yields

$$\begin{aligned}
 r - r^* &= \frac{1/a_1}{\{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet)] + \frac{\mathcal{J}Var_i \mathbf{p}_{t+2} / \mathcal{J}i_t}{2[\mathcal{J}\mathbf{p}_{t+2} / \mathcal{J}i_t]}\}} (\mathbf{p} - \mathbf{p}^*) \\
 &+ \frac{1/a_1 f(\bullet)}{\{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet)] + \frac{\mathcal{J}Var_i \mathbf{p}_{t+2} / \mathcal{J}i_t}{2[\mathcal{J}\mathbf{p}_{t+2} / \mathcal{J}i_t]}\}} \\
 &+ \frac{1/a_1 (\frac{\mathcal{J}Var_i \mathbf{p}_{t+2} / \mathcal{J}i_t}{2[\mathcal{J}\mathbf{p}_{t+2} / \mathcal{J}i_t]})}{\{1 - \mathbf{j}[(\mathbf{p} - \mathbf{p}^*) + f(\bullet)] + \frac{\mathcal{J}Var_i \mathbf{p}_{t+2} / \mathcal{J}i_t}{2[\mathcal{J}\mathbf{p}_{t+2} / \mathcal{J}i_t]}\}} + \mathbf{b}_1 y_t
 \end{aligned} \tag{5.1}$$

Equation (5.1) implicitly defines the optimal level of the nominal interest rate in the nonlinear stochastic model where the uncertainty is only allowed to affect the variance of the target variable. Thus, this is as close as we can get to the linear-quadratic Brainard framework. Since both the left-hand side and the right-hand-side depend on the nominal interest rate, again we have to resort to numerical methods to find the optimal level of the central bank's policy instrument. The results can be found in column 9 of Table 3.1. This column gives the *difference* between the level of nominal rates implied by the rule (5.1) and the nonlinear certainty equivalent rule (3.1). As can be seen from the numbers in this Table the difference is *positive*, implying that in the nonlinear model uncertainty about policy calls for a *higher* rather than a *lower* optimal amount of policy.

6 Summary and Concluding Remarks

In this paper we extended the Svensson (1997a) inflation forecast targeting framework with a convex Phillips curve. Using optimal control techniques we derived an asymmetric policy rule. We found that nominal interest rates according to this rule were higher than under the Svensson forward looking version of the Taylor rule.

Extending the analysis with uncertainty about the output gap we found that our earlier results became even stronger. We found that the uncertainty induced a *further* upward bias in nominal interest rates on top of the effect of the nonlinearity per se.

Also we found that the implications of uncertainty for optimal policy are quite different from either the standard Brainard (1967) analysis, or Svensson's (1997b) extension of his inflation forecast targeting

framework with model uncertainty. More specific, we find that the variability of inflation around the central projection can be reduced by increasing short-term interest rates, calling for a more activist rather than a less activist policy. The implication for policy is that with uncertainty about the output gap (and asymmetries in the output inflation trade-off), cautious policymaking implies a more activist (more aggressive) rather than a less activist (more passive) interest rate policy.

The analysis can be extended in a number of ways. One is to investigate robustness of results with respect to alternative assumptions about inflation expectations. It would be interesting to see whether the same results are obtained with purely model consistent expectations, or a backward and forward-looking components model, or a multiple-regime model with credibility and learning.³⁷

Another is to extend the objective function of the authorities to include an intrinsic weight on output stabilisation. Results can then be contrasted with pure inflation targeting. We leave those issues for further research.

Appendix A The Minimum of Equation (3.2)

In this appendix A we prove that the interest rate differential (3.2) has a local minimum at $(\mathbf{p} - \mathbf{p}^*, y) = (0, 0)$.

The partial derivatives are given by

$$\frac{\mathcal{J}(r_{NL} - r_L)}{\mathcal{J}(\mathbf{p} - \mathbf{p}^*)} = \frac{1}{\mathbf{a}_1} \left[\frac{1}{(1 - \mathbf{j}\Gamma)^2} - 1 \right] \quad (\text{A.1.a})$$

$$\frac{\mathcal{J}(r_{NL} - r_L)}{\mathcal{J}y} = \frac{1/\mathbf{a}_1 f'(\bullet)}{(1 - \mathbf{j}\Gamma)^2} - 1 \quad (\text{A.1.b})$$

where $\Gamma \equiv (\mathbf{p} - \mathbf{p}^*) + f(\bullet) < \frac{1}{\mathbf{j}}$

Hence, it can be easily be seen that if $\mathbf{p} = \mathbf{p}^*$ and $y = 0$, $f(\bullet) = \Gamma = 0$ and $f'(\bullet) = \mathbf{a}_1$ so that (A.1.a) = (A.1.b) = 0, and $(\mathbf{p} - \mathbf{p}^*, y) = (0, 0)$ is a *stationary point*.

The second derivatives and the cross partials are

$$\frac{\mathcal{J}^2(r_{NL} - r_L)}{\mathcal{J}(\mathbf{p} - \mathbf{p}^*)^2} = \frac{2\mathbf{j}}{\mathbf{a}_1(1 - \mathbf{j}\Gamma)^3} \quad (\text{A.2.a})$$

³⁷ For an interesting analysis that builds on a trade-off between caution and learning (by experimentation) in policy see Wieland (1998).

$$\frac{f^2(r_{NL} - r_L)}{f_y^2} = \frac{1/a_1[f'(\bullet)(1-j\Gamma) + (2j/a_1)[f'(\bullet)]^2]}{(1-j\Gamma)^3} \quad (\text{A.2.b})$$

$$\frac{f^2(r_{NL} - r_L)}{f(\mathbf{p} - \mathbf{p}^*)f_y} = \frac{f^2(r_{NL} - r_L)}{f_y f(\mathbf{p} - \mathbf{p}^*)} = \frac{2jf'(\bullet)}{a_1(1-j\Gamma)^3} \quad (\text{A.2.c})$$

Now because (A.2.a) is positive, and the determinant

$$\begin{vmatrix} \frac{f^2(r_{NL} - r_L)}{f(\mathbf{p} - \mathbf{p}^*)^2} & \frac{f^2(r_{NL} - r_L)}{f(\mathbf{p} - \mathbf{p}^*)f_y} \\ \frac{f^2(r_{NL} - r_L)}{f(\mathbf{p} - \mathbf{p}^*)f_y} & f(y)^2 \end{vmatrix} \bigg|_{\substack{\mathbf{p} - \mathbf{p}^* = 0 \\ y=0}} = \begin{vmatrix} 2j/a_1 & 2j \\ 2j & 2j(1+a_1) \end{vmatrix} = \frac{4j^2}{a_1} \quad (\text{A.3})$$

evaluated at the stationary point is positive, the surface near (0,0) is in the shape of a ‘bowl’ and we have a local minimum.

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